An Analysis of Estimates of the Inventory Holding Component of the Bid-Ask Spread

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We examine the performance of three spread decomposition models which provide estimates of the inventory holding component of the bid-ask spread. As benchmarks for the analysis, we use three order imbalance metrics as well as the percentage spread. We compare the inventory holding cost estimates of these models with variables reflecting conditions expected to impact the level of inventory cost. The models studied produce mixed results. Inventory holding costs correlate with various measures of trading frequency, volatility and the availability of traded options. Two of the models report the expected relationships with a significant number of explanatory variables yet their inventory cost estimates vary widely. Additional concerns relate to the possibility that these models are merely weak reflections of the spread itself since the variables found to impact inventory holding cost are also well known to be strong determinants of the overall spread.

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1. Introduction

The market microstructure literature offers a series of models aimed at decomposing the bid-ask spread, typically into three components: order processing, adverse selection (or asymmetric information), and inventory-holding.[2] The order processing cost component represents a fee charged by market makers for standing ready to match buy and sell orders, the adverse selection component is the compensation to market makers for taking the risk of dealing with traders who may possess superior information, and the inventory holding component is the compensation for a dealer holding a less diversified portfolio. While much research has concentrated on the adverse selection cost, the order processing and inventory costs have received little direct attention and are frequently reported combined into a single result. Consequently, we know little about how spread decomposition models perform relative to each other nor how well they perform against various proxies for the presence of inventory costs.

In this study we focus directly on the inventory holding cost component of the spread. We examine the performance of three spread decomposition models in isolating and measuring inventory cost using various indicators of conditions affecting inventory costs and proxies for the presence of inventory costs.

Using both univariate correlation measures and ordinary least squares regression techniques, we find mixed results in terms of both significance and consistency with predictions. Inventory holding costs are generally positively related to volatility and trade size and negatively related to measures of trading frequency and the availability of traded options. Overall, the results for two of the three models are quite similar to those for the spread itself raising doubts as to the added value of these models in estimating inventory costs.

The paper proceeds as follows: Section 2 discusses the background on inventory holdings costs. Section 3 discusses the data and methodology. Section 4 presents the analysis and results, and Section 5 concludes.

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[2] See: Copeland and Galai (1983), Glosten and Milgrom (1985), Glosten (1987), Glosten and Harris (1988), Hasbrouck (1988), Ho and Stoll (1981, 1983), and Stoll (1978, 1989).

2. Background

The order processing component of the spread may be considered the cost of doing business, and while it will certainly vary over time, it can be expected to be relatively stable over the short-term. Intraday changes in the spread must then necessarily be driven by the remaining two components: adverse selection and/or inventory holding. Since adverse selection is the more intuitively obvious driver of spread change, the literature has focused on estimation and analysis of this element. The cost of holding assets in inventory and attempting to correct a less than optimal inventory level is a more subtle, yet potentially important determinant of a market maker's quote placement tactics.

Naik and Yadav (2002) and Bessembinder (2002) suggest that if market makers perceive that competitive quotations will attract orders, then reductions (increases) in inventory should lead to posting of more aggressive quotations at the bid (ask) to attract customer sell (buy) orders and restore inventory. Bessembinder finds a significant relationship between inventory effects and quote placement for U.S. equities both on and off the NYSE, but substantially more for the NYSE. Additionally, Hansch, Naik, and Viswanathan (1998) document that inventory affects the quote behavior of London Stock Exchange dealers**.**

Despite this documented significance, little is known concerning the performance of spread decomposition models relative to the inventory component. In addition, various models report a broad range for inventory costs. For instance, Stoll (1989), using a sample of NASDAQ stocks, finds that inventory holding costs account for 10% of the spread. Huang and Stoll (1997) using a sample of 20 of the largest and most active NYSE stocks, find inventory costs represent 28.7% of the spread. Bollen, Smith, and Whaley (2004), using NYSE stocks, find that the inventory holding cost is the largest component of the spread, with estimates of 29.28%, 32.35%, and 44.69% for March 1996, April 1998, and December 2001, respectively. While these studies involve different data sets (stocks, markets and time periods), such a considerable variation is worthy of further investigation.

A wide disparity in component cost estimates is also reported in studies of the adverse selection component. For example, Flannery, Kwan and Nimalendran (2000) studying the opacity of banks, find that the adverse selection models of George, Kaul and Nimalendran (1991) (as modified by Neal and Wheatley, 1998) and Lin, Sanger and Booth (1995) produce quite varied results. Van Ness, Van Ness, and Warr (2001), using five spread decomposition models and a sample of NYSE stocks, also report inconsistent and inconclusive results in adverse selection component estimation.

Since spread components are estimated as percentages of the total spread, wide differences in the estimation of any one component necessarily calls into question the ability of a model to accurately isolate any of the three components.

We extend the study of spread decomposition model performance by focusing on the ability of three models to estimate the inventory holding component of the bid-ask spread. To analyze the performance of these models, we compare their inventory component estimates with variables reflecting conditions impacting the presence or level of inventory costs. These impact variables include the number of shareholders, trading frequency, volatility, volume, and options listings. In addition, we use the percentage spread and three order imbalance metrics as benchmarks for performance comparisons.

While our primary focus is on the performance of the inventory holding component estimates, our analysis is necessarily a multi-pronged test of the spread decomposition models and of the ability of our benchmark and proxy variables to measure conditions that should indicate the presence or impact the level of inventory holding costs in the spread. Specifically, we address each of the following areas in developing our analysis of the decomposition model estimates:

1. Do our order imbalance variables accurately measure inventory imbalance conditions?
2. Are the inventory holding cost estimates of our three models consistent with expectations and with the indications of our impact and benchmark variables?
3. Do the model estimates improve on the spread itself as an indicator of inventory holding costs?

3. Data and Method

3.1 Data

Intraday trade and quote data are obtained from the NYSE TAQ database for August 2005. Option data is from the Standard and Poor's Stock Guide. We use only NYSE trades and exclude stocks with characteristics which might differ from ordinary equities such as assets in the following categories: certificates, ADRs, REITs, closed-end funds, preferred stocks, companies incorporated outside of the U.S., and financial services firms. We also eliminate observations meeting any of the following criteria:

- A trade or quote is out of sequence, recorded before the open or after the closing time; 9:30-16:00 (outside of regular trading hours).
- The bid price is greater than the ask price.
- The stock price is less than \$5.
- The stock's average daily number of trades is less than 50.

Finally, as in Huang and Stoll (1997), we collapse all trades at the same price and quotes into a single trade (bunching procedure).[3] Our final sample of NYSE firms includes 1,175 stocks.

3.2 Inventory Holding Component Models

Several approaches are used to estimate the components of the bid–ask spread. In one group of models, inferences about the bid-ask spread are made from the serial covariance of transaction prices (George, Kaul, and Nimalendran, 1991; Roll, 1984; and Stoll, 1989). In the second group, the components are inferred by relating changes in prices to transaction size and to the direction of trades (buyer or seller initiated) (Glosten and Harris, 1988; Hasbrouck, 1988, 1991; Huang and Stoll, 1997; and Madhavan, Richardson, and Roomans, 1997).

Since we are focused on the inventory premium, we test only decomposition models that have a clearly defined inventory holding component: Stoll (1989), Huang and Stoll (1997), and Bollen, Smith, and Whaley (2004). For each model, we initially compute the inventory holding component as a percentage of the spread. In order to control for price, we also express the dollar inventory holding cost as a percentage of the stock price, providing a measure of the inventory holding cost of trading for a given dollar.

3.2.1 Stoll's (1989) Model (S89)

Stoll (1989) examines the quoted and realized spreads to model the behaviour of the spread and infer the components of the spread. He introduces two specific parameters integral to his model: the probability of a price reversal (π) and the value of a price continuation as a fraction of the spread (δ). He decomposes the spread into three components - order processing, inventory holding, and adverse selection - by examining the covariance of the trade returns and the covariance of returns defined over the quoted bid-ask prices. Stoll assumes that the market is efficient such that price changes are independent of both current and past information. Under this assumption of market efficiency, the serial covariance of price changes due to the spread may be inferred from observed price changes.

Stoll calculates the three spread components from the slope coefficients of regressions of the serial covariance of the percentage price change of the bid-ask spread:

$$\mathrm{cov}_T = a_0 + a_1 S^2 + u \tag{1}$$

$$\mathrm{cov}_Q = b_0 + b_1 S^2 + v, \tag{2}$$

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[3] Huang and Stoll (1997) use this bunching procedure since, as they argue, 'a large order may be executed at a single price but be reported in a series of small trades' and 'a single large limit order may be executed at a single price against various incoming market orders', which may cause order flow correlation to appear greater than it actually is. However, using this procedure, the authors note, may overcorrect this problem.

where S^2 is the squared value of the average quoted proportional spread (i.e., the difference between the ask and bid quotes divided by the average of these quotes); cov_T is the serial covariance of transaction price changes; cov_Q is the serial covariance of changes in bid (or ask) quotes; a_0 and b_0 are constant; a_1 and b_1 are the coefficient of spread, and u and v are random error terms. The inventory holding cost model predicts that both covariance's are negative because the probability of a price reversal (π) is higher than 0.5 and δ =0.5.

Given a_1 and b_1, Stoll solves for intermediate values, ∂, the size of a price continuation as a fraction of the spread, π, the probability of a price reversal, and $(1-\pi)$, the probability of a price continuation, from two auxiliary equations[4]:

$$a_1 = \partial^2(1-2\pi) - \pi^2(1-2\partial), \tag{3}$$

and

$$b_1 = \partial^2(1-2\pi). \tag{4}$$

The components of bid-ask spreads are then determined as follows:

$$\text{Adverse Selection Cost} = \left[1-2(\pi-\partial\right] \tag{5}$$

$$\text{Inventory Holding Cost} = 2(\pi-0.5) \tag{6}$$

$$\text{Order Processing Cost} = \left(1-2\partial\right) \tag{7}$$

3.2.2 The Huang and Stoll (1997) model (HS)

Huang and Stoll (1997) propose two-way and three-way spread decomposition models. The two-way decomposition model does not separate the adverse selection and inventory holding components. The three-way decomposition model allows for the identification of all three components: adverse selection, order processing, and inventory holding costs.

The three-way decomposition of the spread is based on induced negative serial correlation in both trade flows (Q_t) and quote changes (ΔM_t) and can be used to identify the inventory holding cost component. The HS model serial correlation in trade flows given by the conditional expectation of the trade indicator at time $t-1$, given Q_{t-2}, $E(Q_{t-1} \mid Q_{t-2})$, can be expressed as:

$$E(Q_{t-1} \mid Q_{t-2}) = (1-2\pi)Q_{t-2} \tag{8}$$

where π is the probability that the trade at t is opposite in sign to the trade at $t-1$, $Q_{t-1}(Q_{t-2})$ is a buy-sell trade indicator at time $t-1$ ($t-2$). With π different from one-half, and equation (8) known to the market, the change in fundamental value will be given by

$$\Delta V_t = \alpha\frac{S}{2}Q_{t-1} - \alpha\frac{S}{2}(1-2\pi)Q_{t-2} + \varepsilon_t, \tag{9}$$

where ΔV_t is the total change in the unobservable fundamental value of the stock in the absence of transaction costs between time $t-1$ and t, and is determined just prior to the posting of the bid-ask quotes at time t and $t-1$, S_t is the posted spread just prior to the transaction, and $\alpha\frac{S}{2}Q_{t-1}$ reflects the private information revealed by the last trade, as in Copeland and Galai (1983) and Glosten and Milgrom (1985). The public information component is captured by ε_t.

Huang and Stoll estimate the components of the spread directly from the following equation:

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[4] If a transaction at the bid is followed by another transaction at the bid (a continuation), the price change is - ∂S, where $0 \prec \partial \prec 1$. If a transaction at the bid is followed by transaction at the ask, the price change is $(1-\partial)S$, a reversal.

$$\Delta M_t = (\alpha + \beta)\frac{S_{t-1}}{2}Q_{t-1} - \alpha\frac{S_{t-2}}{2}(1 - 2\pi)Q_{t-2} + e_t \qquad (10)$$

where Q_t is the buy-sell indicator for the transaction price, P_t . M_t is the midpoint of the quote that prevails just before the transaction at time t in the above equation. The estimation of α and β are percentages of the half-spread attributable to adverse selection and inventory holding costs, respectively. Since α and β are stated as proportions, the order processing component is equal to (1-α-β).

3.2.3 The Bollen , Smith, and Whaley (2004) model (BSW)

The uniqueness of the Bollen, Smith, and Whaley (2004) model rests in the use of the option approach to identifying the determinants of the bid-ask spread. The market maker's spread includes a premium to cover expected inventory-holding costs, independent of whether the trade is initiated by an informed or an uninformed trader. In the BSW model, the authors assume that the length of time a stock is held in inventory is known and short. BSW also assume that the risk-free rate and the expected change in the true price of the stock are equal to zero for this short holding period.

In the absence of a viable hedging instrument, the market maker faces inventory –holding price risk for which he will demand compensation. BSW show that the value that the market maker is willing to accept as compensation for his inventory holding cost is equal to the value of an at-the-money option with maturity given by the time that the stock is held. So, the inventory holding premium (IHP) is defined as:

$$IHP = S\left[2N(0.5\sigma)\sqrt{t}) - 1\right] \qquad (11)$$

where, S is the stock price, σ is the standard deviation of the security return, t is the time to maturity and N (.) is the cumulative unit normal density function.

Given that t is unknown, the expected inventory holding premium, E (IHP) is shown to be:

$$E(IHP) = S\left[2N\left\{0.5\sigma E(\sqrt{t})\right\} - 1\right] \qquad (12)$$

In the implementation of the model, E($\sqrt{t}$) is computed as the average of the square root of the time between trades.

When asymmetric information costs are considered, a distinction is made between an informed and an uninformed trader. BSW demonstrate that when the trade is uninformed, the compensation demanded by the dealer equals the value of a slightly out-of-the money (OTM) call option with an exercise price equal to the ask price.[5] If the investor is informed, he knows that the true price of the stock will be above the ask price and the value of the compensation to the market maker will be equal to the value of a slightly in-the-money call option. Hence, whether the trader is informed (I) or uninformed (U), the market maker requires the following compensation for bearing inventory risk:

$$IHP_i = S_i N\left(\frac{\ln(\frac{S_i}{X})}{\sigma\sqrt{t}} + 0.5\sigma\sqrt{t}\right) - XN\left(\frac{\ln(\frac{S_i}{X})}{\sigma\sqrt{t}} - 0.5\sigma\sqrt{t}\right) \qquad (13)$$

where S is the true stock price at the time that the market maker opens his position, X is the exercise price of the option, σ is the standard deviation of the security returns, t is the time until the offsetting order, and $N\left(.\right)$ is the cumulative unit normal density function, and I = U, I.

3.3 Variables

Spread decomposition models, in which inventory-holding costs are one of the components, rely on the assumption that uncertainties in order flow can result in inventory problems for market

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[5] The compensation required by the market maker may also be valued through a put option in which case the exercise price will equal the bid price.

makers. Since order flow does not always balance due to unpredictable supply and demand, market makers often carry inventory in the course of supplying liquidity and, hence, bear risk. Amihud and Mendelson (1980) and Ho and Stoll (1980) state that the greater the liquidity risk, the more difficult it is for the market maker to return to his desired inventory level.

The presence and level of an inventory holding component in the spread depends on two conditions: the existence of an inventory imbalance and the ease with which the market maker is able to hedge or rebalance his position without significant adverse price movement.

Our variables of interest fall into two broad categories: benchmark variables and impact variables. The latter category measures conditions expected to directly impact the level of the inventory holding cost component. In addition, to control for differences in trading price, we include price as a variable.

3.3.1 Benchmark Variables

Variables in this category serve two distinct purposes in our analysis. One objective is to measure the presence of an imbalance situation while the second is to determine if the inventory holding cost estimates perform better than the total spread in reflecting the presence of inventory holding costs.

a) Order Imbalance Variables

The need to increase the inventory component of the spread may be driven by two overall market conditions, independently or working together. The most obvious is an inventory imbalance. Since we cannot observe a market maker's individual inventory position, we employ measures of overall market imbalance in each stock as proxies. The order imbalance variables are selected to measure the presence of an unbalanced inventory condition.

According to Stoll (1978a) and Ho and Stoll (1981), order-flow imbalances give rise to the inventory holding cost component of the bid-ask spread. The process of equilibrating order imbalances may cause the market maker's inventory position to deviate from optimal levels, with larger deviations leading to larger inventory holding costs.

Bessembinder (2002) constructs the relative order imbalance for each market using the Ellis, Michaely, and O'Hara (2000) algorithm[6]. He signs trades as buyer or seller initiated and determines the imbalance based on the accumulated difference between customer buy and customer sell trades from the opening trade on a particular market. Chordia, Roll, and Subrahmanyam (2002), who sign trades using the Lee and Ready (1991) algorithm, calculate their measure of the daily order imbalance in each stock using the number of buys and sells, the quantity bought and sold and the dollar volume bought and sold. In later work, Chordia and Subrahmanyam (2004) follow a similar approach, but elect to exclude the dollar volume imbalance.

As benchmarks for our analysis we use Bessembinder's (2002) and Chordia, Roll, and Subrahmanyam's (2002) metrics for order imbalances. Each transaction is designated as either buyer or seller initiated according to the Lee and Ready (1991) method (quote rule for trades away from the quote mid-point, tick rule for trades at the mid-quote)[7]:

- If a trade at t executes above (below) the prevailing mid-quote then q_t=1 (q_t=-1);
- If a trade at t executes at the mid-quote and P_t=P_{t-1} then q_t=q_{t-1}.

For each stock we calculate three measures of order imbalance. As there will potentially be both long and short position changes in specialists'/market makers' inventory positions, we employ the absolute value of our order imbalance variables:

OIBNUM = the absolute value of the number of buyer-initiated less the number of

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[6] He also uses the Lee and Ready (1991) algorithm, with no significant deviations in reported results.

[7] Lee and Ready (1991) recommend comparing trade prices to quotes in effect five seconds before the trade report time to allow for delays in trade reporting. Bessembinder (2002) finds that trade direction and rates of price improvement are best assessed when making no adjustment for trade report lags. However, Bessembinder and Venkataraman (2009) note that Bessembinder (2003) recommends using quotation midpoints in effect five seconds prior to the trade report time as a proxy for the true underlying price. We elect to use no trade lag in our analysis.

seller-initiated trades on day t.

OIBSH =the absolute value of the buyer initiated shares purchased less the seller-initiated shares sold on day t.

OIBDOL = the absolute value of the buyer-initiated dollars paid less the seller-initiated dollars received on the day t.

b) The Bid-Ask Spread

Since inventory cost is a portion of the spread, estimates of that component should perform as least as well as the spread itself in our comparison. Following Van Ness, Van Ness, and Warr (2001) we include the percentage bid-ask spread as an additional benchmark in the analysis.

3.3.2 Impact Variables

The need of the market maker to increase his spread to reflect higher inventory holding costs reflects not only the presence of an imbalance but also conditions making it difficult or costly to return to an optimal inventory level. Conversely, if market conditions are such that he can easily offset any holding risk, inventory holding costs may be reduced.

We employ seven variables to measure these conditions. Trade size and volatility are expected to increase the inventory holding component while dollar and share volume, number of trades, shares outstanding and the availability of traded options are expected to reduce inventory holding costs.

a) Trade Size

Theoretical and empirical evidence suggests that order flow affects transactions costs by changing a specialist's cost of holding inventory, as well as by providing signals about security value. Lin, Sanger, and Booth (1995) suggest that the inventory of a specialist may be influenced by dimensions of order flow. Larger trades lead to inventory imbalances, driving higher inventory costs, thus we include trade size in our study.

b) Volatility

Inventory explanations of spreads predict a positive correlation between the spread and volatility, such as is found by Tinic (1972), Stoll (1978b, 2000), and Ho and Stoll (1981, 1983), Bessembinder (1994), and Chordia and Subrahmanyam (2004). Notably, Wyart, Bouchaud, Kockelkoren, Potters, and Vettorazzo (2008) extend these findings to demonstrate a strong positive correlation between the spread and the volatility per trade (emphasis in the original) rather than per unit of time[8]. Ho and Stoll (1981) demonstrate that the more volatile the stock price, the more the market maker is exposed to the risk of adverse price movements. Tinic (1972) employs the standard deviation of price as a measure of inventory price risk. Interestingly, Prucyk (2005) finds that the inventory component of the spread increases both when volatility is increasing and when it is decreasing, though less significantly when decreasing. We use the standard deviation of the quote midpoint to capture intraday volatility in the true price of the stock.

c) Trading Volume

The inventory holding component of the spread models predict that as the liquidity of a stock increases, the compensation required by the specialist through the spread declines, resulting in a negative relationship between trading volume and the spread. If trading volume is low, specialists will find it difficult to adjust their inventory levels and will increase their spreads to compensate. Stoll (1978b, 1989) uses dollar volume as a proxy for the length of the market maker's holding period (one of the factors influencing inventory-holding costs). Stoll (1989) uses trading volume as a proxy for the risk of holding non-optimal inventory levels. Duan and Jung-Chu (2007), using the Bollen, Smith, and Whaley (2004) model to study ETF spreads, confirm that when trading volume increases, inventory holding costs decline. Based on these findings, we include both dollar volume and daily

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[8] Wyart, M., J-P Bouchaud, J. Kockelkoren, M. Potters, and M. Vettorazzo, 2008, Relation between bid-ask spread, impact and volatility in order-driven markets, *Quantitative Finance* 8, p 41

share volume in our analysis.

d) Trade Frequency

As previously noted, Lin, Sanger, and Booth (1995) suggest that the inventory of a specialist may be influenced by dimensions of order flow, such as the frequency of trades and the timing of trades.

Demsetz (1968) argues that price concessions charged for immediate transactions will be inversely related to a particular security's trading activity. In an actively traded market a dealer can correct sub-optimal inventory positions with greater speed. Demsetz uses the number of shareholders as a proxy for long-run trading activity, arguing that the number of shareholders is a direct proxy for the transaction rate. The higher the transaction rate, the lower the cost of waiting and the lower the bid-ask spread. In that spirit, we include the number of trades and number of shares outstanding in our analysis.

e) Option Listings

Option listings have a beneficial impact on the underlying stock. Ross (1976) and Hakansson (1982) show that options improve the efficiency of incomplete asset markets by expanding the opportunity set facing investors. Additionally, Bollen et al (2004) show that option listings reduce spreads and improve liquidity in the underlying stock by reducing the inventory costs of the market maker since options provide a mechanism for hedging their inventory positions. We include the existence of traded options on each stock in our analysis.

4. Results and Analysis

In this section, we present summary statistics on our data sample and discuss inventory holding component estimates and their relations to each other as well as to the variables predicted to impact inventory holding costs. We then examine these relations in an ordinary least squares framework.

4.1 Summary Statistics

Table 1 presents summary inventory holding component statistics for the sample, computed using the three models under study. Panel A details raw estimated inventory holding costs as proportions of the spread. The Stoll model (S89) produces the lowest average cost at 0.0641 but with an implausible range of -0.1790 to 10.257. The Huang and Stoll model (HS) produces the highest estimate at 0.2738, again with an impossible range of -3.7520 to 152.0550. The Bollen, Smith and Whaley model (BSW) produces an average inventory cost of 0.2152 with a range of 0.0491 to 0.8225. With two models producing impossible ranges and significant variation in the three model's estimates, there is clearly sufficient motivation to analyze which, if any, of these models produces estimates usable in practice. Since the ranges for the S89 and HS models (MIN, MAX) are theoretically impossible, in all future analysis, we use only the components that are between zero and one (these constrained numbers are presented in Panel B of Table 1).[9]

Panel B of Table 1 restricts the sample to theoretically plausible values, between 0 and 1, reducing the sample to 598 observations for the HS model and 1127 observations for the S89 model. The mean estimate of the inventory holding component for S89 drops from 0.0641 to 0.0530 after eliminating 48 observations. For the HS model, constraining the estimates eliminates more than half of the observations while reducing the inventory holding cost estimate by almost one half, which could severely impact the applicability of this model.

Panel C presents the cost of inventory holding as a percentage of share prices, with the percentage constrained to values between 0 and 1. The average inventory holding premium for a $10 stock is $0.028 for the S89 model, $0.094 for the HS model, and $0.062 for the BSW model, again, a wide range with the HS estimate over three times S89's.

Panel D and Panel E present the probability of a price reversal for the S89 and HS models,

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[9] Van Ness, Van Ness, and Warr (2001), among others, also find that the Huang and Stoll (1997) model produces a large number of theoretically implausible estimates.

respectively. The BSW model does not produce a comparable estimate. Note that even with all estimates considered, the Stoll model's probability of a price reversal centers closely around 0.5 (mean = 0.5021 with a range of 0.4970 to 0.5025). Recall that the Stoll model assumes the probability of a price reversal to be greater than 0.5.

Table 1
Summary Statistics of Inventory Holding Premium

	Mean	Median	Std. Deviation	Min.	Max.	Observations
Panel A: Components as a Proportion of the Spread						
S89	0.0641	0.0068	0.4584	-0.1790	10.2567	1,175
HS	0.2738	0.0032	4.6504	-3.7520	153.0550	1,175
BSW	0.2152	0.1995	0.0843	0.0491	0.8255	1,175
Panel B: Inventory Holding Components >0 and <1 as a Proportion of the Spread						
S89	0.0530	0.0071	0.0877	0.0009	0.9886	1,127
HS	0.1428	0.0920	0.1606	0.0008	0.9657	598
BSW	0.2152	0.1995	0.0843	0.0491	0.8255	1,175
Panel C: Dollar Inventory Holding Components Divided by Price (Constrained as in Panel B) and ×100						
S89	0.0028	0.0016	0.0030	0.0000	0.0175	1,127
HS	0.0094	0.0096	0.0053	0.0002	0.0184	598
BSW	0.0062	0.0057	0.0044	0.0000	0.0199	1,175
Panel D: Probability of Price Reversal for Stoll (1989)						
All Estimates	0.5021	0.5021	0.0001	0.4970	0.5025	1,175
Plausible Estimates	0.5021	0.5021	0.0001	0.4908	0.5209	1,127
Implausible Estimates	0.5024	0.5026	0.0001	0.5016	0.5026	48
Panel E: Probability of Price Reversal for Huang and Stoll (1997) Model						
All Estimates	0.5173	0.5050	0.1299	0.1492	0.9654	1,175
Plausible Estimates	0.5229	0.5443	0.0790	0.5192	0.5310	598
Implausible Estimates	0.6150	0.5915	0.0955	0.4737	0.9654	577

Notes: This table presents summary statistics of the 1,175 stocks listed on the NYSE in August 2005 and included in the sample. For each stock, the inventory holding premium component of the spread is computed using three different spread decomposition models. The models are Stoll (1989) (S89), Huang and Stoll (1997) (HS), and Bollen, Smith, and Whaley (2004) (BSW).

The HS model (Panel E), with all estimates included, produces a reasonable average probability of a price reversal of 0.5173 but with a range of 0.1492 to 0.9654. This high deviation is eliminated with the implausible estimates, resulting in a mean of 0.5229 and a range of 0.5192 to 0.5310. While the issue of unusable estimates is resolved, the necessity to eliminate close to fifty percent of the observations clearly compromises the model's strength. Again, recall that the HS model assumes that the probability of a price reversal is not equal to 0.5.

As two of our models produce theoretically implausible estimates for some stocks, we will present many of our results two ways. Results with only the theoretically plausible estimates for each model, henceforth the "constrained sample," will include a different number of observations for each model. Results in which all of the models have theoretically plausible estimates for the same stocks (the stocks have to have theoretically plausible estimates for all three models), yields a subsample of 566 observations.

Table 2 presents summary statistics for all 1,175 stocks in our sample for our selected variables of interest. Panel A describes the average firm as having 982 trades per day (NTS), an average trade size of 468 shares (TSIZE) and an average trade price of $35.63 (TPRICE). The range for all variables demonstrates that the stocks in our sample cover a wide range over every dimension. Panel B of Table 2 presents descriptive statistics for the absolute values of the order imbalance variables. Again, the range for each of the imbalance variables is sufficiently broad to insure sufficient variation in the stocks in our sample.

Table 2
Summary Statistics: Other Variables

	Mean	Median	Std. Deviation	Min.	Max.	Observations
Panel A. Other Variables						
NTS	982	774	772	55	5,397	1,175
TSIZE	468	363	350	140	4,054	1,175
VOL	628,412	287,104	1,141,020	14,691	17,058,383	1,175
DOLVOL	24,007,932	8,917,999	44,606,070	220,954	553,393,640	1,175
SDMID	1.03	0.75	1.28	0.04	16.02	1,175
TPRICE	35.63	31.87	22.38	5.34	102.98	1,175
OUTSHR	175,253,234	76,700,000	284,626,594	9,935,150	3,178,533,000	1,175
SPREAD	0.0387	0.0295	0.0346	0.0100	0.7125	1,175
PSPREAD	0.0013	0.0010	0.0011	0.0002	0.0106	1,175
Panel B. Order Imbalances in terms of number of shares, dollars, and number of transactions						
OIBSH	591,574	250,052	1,192,828	12,535	17,712,069	1,175
OIBDOL	22,425,464	7,622,502	45,138,652	242,244	565,225,035	1,175
OIBNUM	1,053	679	1,222	43	11,718	1,175

Notes: This table presents descriptive statistics for the 1,175 stocks in the sample. Panel A includes the following: NTS is the average number of trades per day; TSIZE is the average trade size; VOL is the total daily share volume; DOLVOL is the total dollar volume; SDMID is the standard-deviation of the midpoint of the quoted spread; TPRICE is the average trade price; OUTSHR is the number of outstanding shares; SPREAD is the average daily bid-ask spread; PSPREAD is SPREAD divided by trade price. Panel B presents descriptive statistics for the three order imbalance variables: OIBNUM is the order imbalance in terms of number of trades. OIBDOL is the order imbalance in terms of dollars. OIBSH is the order imbalance in terms of number of shares.

While Tables 1 and 2 focus on raw summary statistics, two issues arise causing early concern. First, the wide variation in the inventory holding cost estimates – both raw and constrained – suggest that the three models, using the same data set, are measuring something quite different in the spread. Second, the high number of implausible estimates generated by the Huang and Stoll model raises a flag as to its viability.

4.2 Correlations

Table 3 presents the Spearman rank correlations between the estimated inventory holding components, the three measures of order imbalance, and the percentage spread for the total sample of 1,175 stocks. Table 4 presents the same analysis using only the subsample of 566 stocks. In both tables, Panel A uses dollar terms to express the inventory holding components measured from the three models. Panel B presents the Spearman correlations after scaling by price.

Looking at Tables 3 and 4 together, the results in all four panels are generally consistent in terms of the direction of the correlation (direct or inverse), with some differences in terms of strength and significance of correlation. To facilitate discussion of these results, we focus on Table 3 Panel A and Table 4 Panel B.

In both these panels, all correlations are statistically significant, providing the strongest results for discussion and evaluation. In both panels, all correlations between the spread and all three inventory holding components are positive as expected and significant, with the strength of the correlations stronger in Table 4 for the S89 and HS models and only slightly weaker for the BSW model. The correlations between the three model results are all positive, statistically significant, and stronger in Table 4, reflecting the result of eliminating unusable estimates. The positive correlations between the three model estimates does suggest that they are in some way measuring similar elements of the spread. It should be noted that the relationships between the HS model and the spread and between the HS model and the other model estimates are the weakest, and, while they improve substantially with the reduced subsample, they remain the lowest of the group.

All three order imbalance variables are strongly and significantly positively related to each other. As predicted, these three benchmark variables appear to be measuring a similar market condition. While the correlations do decline with the reduced sample, they are still strong at above 75%.

Table 3
Spearman Rank Correlation Coefficients for Inventory Holding Components

	SPREAD	S89	HS	BSW	OIBSH	OIBDOL
Panel A. Dollar Estimates						
S89	0.7133***					
HS	0.2095***	0.1420**				
BSW	0.8412***	0.4886**	0.1537**			
OIBSH	-0.7411***	-0.7161***	0.1437***	-0.5238***		
OIBDOL	-0.5038***	-0.7347***	0.0799**	-0.2443***	0.9036***	
OIBNUM	-0.6338***	-0.7487***	0.1119***	-0.3937***	0.9473***	0.9551***
N	1,175	1,175	1,175	1,175	1,175	1,175
Panel B. Scaled by Price						
S89	0.7483***					
HS	0.1042***	0.1484**				
BSW	0.3753***	0.4909**	0.3419**			
OIBSH	-0.7353***	-0.4766***	0.0918**	-0.0447		
OIBDOL	-0.8580***	-0.6662**	-0.0873**	-0.3705***	0.9027***	
OIBNUM	-0.8193***	-0.5888***	-0.0018	-0.2165***	0.9480***	0.9552***
N	1,175	1,175	1,175	1,175	1,175	1,175

Table 4
Spearman Rank Correlation Coefficients for Inventory Holding Components

	SPREAD	S89	HS	BSW	OIBSH	OIBDOL
Panel A. Dollar Estimates						
S89	0.5150***					
HS	0.4996***	0.2633**				
BSW	0.7113***	0.1863**	0.3877***			
OIBSH	-0.5308***	-0.5341***	-0.3051***	-0.2632***		
OIBDOL	-0.2975***	-0.6399***	-0.1831***	0.0084	0.8968***	
OIBNUM	-0.4308***	-0.6238***	-0.2460***	-0.1219**	0.9280***	0.9473***
N	566	566	566	566	566	566
Panel B. Scaled by Price						
S89	0.8928***					
HS	0.5104***	0.4250***				
BSW	0.8245***	0.7527***	0.4692***			
OIBSH	-0.2374***	-0.1162***	-0.2631**	-0.2863***		
OIBDOL	-0.6506***	-0.5343***	-0.4031***	-0.6002***	0.7910***	
OIBNUM	-0.2795***	-0.1648***	-0.2818***	-0.3106***	0.9462***	0.7801***
N	566	566	566	566	566	556

Notes: tables 3 and 4 presents the Spearman rank correlations for the three inventory holding premium models and the three measures of order imbalances. We use the models of Stoll (1989), Huang and Stoll (1997), and Bollen et al. (2004) to compute the inventory holding cost components of the full sample, including the implausible estimates of Huang and Stoll (1997) and Stoll (1989). In Panel A, the inventory holding cost components are expressed as raw dollar costs. In Panel B, the components are expressed as a percentage of the stock price. *** Statistically significant at the 0.01 level. ** Statistically significant at the 0.05 level. * Statistically significant at the 0.1 level.

The major concern with the Tables 3 and 4 results is with the relationship between the order imbalance variables and the spread and inventory holding cost estimates. In both tables these correlations are significant and unexpectedly negative. The sole exception is the HS model in Table 3 where the correlations are positive but shift to all negative in the subsample results (Table 4). The order imbalance variables are designed to capture the presence of an imbalance condition while the spread and inventory holding cost estimates reflect the reaction of the market maker to an imbalance. The relationship between the presence of an imbalance and an associated cost need not be strong as other conditions could mitigate the market maker's response, but intuitively it should at least be positive. These results call into question the applicability of the order imbalance variables as benchmarks for our analysis.

Table 5 presents the Spearman rank correlations between the estimated inventory holding components and our impact variables. Panel A provides results for the constrained sample, while Panel B details results for the subsample of 566 stocks.[10] Following the empirical work on determinants of the bid-ask spread (Benston and Hagerman, 1974; Branch and Freed, 1997; Brennan and Subrahmanyam, 1995), we express both our dependent and independent variables in natural logarithms. Specifically following Benston and Hagerman (1974), who examine several functional forms, we determine that the log linear form is the most suitable for the data based on assumptions related to the intended use of ordinary least squares analysis (for example, the expected zero value of the error terms, uncorrelated errors, and the constant variance of the errors across observations).

Table 5
Spearman Rank Correlation Coefficients for Inventory Holding Components and other variables

Panel A. Different sample size for each group				
	EXP	S89	HS	BSW
LNTS	-	-0.7243***	-0.1684**	0.2833***
LTSIZE	+	0.3880**	0.1782**	0.0247
LVOL	-	-0.6535**	-0.2098***	0.2110***
LOUTSHR	-	-0.6555***	-0.1524	0.8350**
SDMID	+	0.7769**	0.0086	0.3396
N		1,127	598	1,175
Panel B. The same sample size for each group				
LNTS	-	-0.6152**	-0.1545**	0.3612**
LTSIZE	+	0.0568**	0.1690**	0.0100
LVOL	-	-0.4916**	-0.1936**	0.2845**
LOUTSHR	-	-0.4651**	-0.1374**	0.1464**
SDMID	+	0.2739**	0.0032	0.3064***
N		566	566	566

Notes: This table presents the Spearman rank correlations for the three inventory holding premium models and the impact variable. We use the models of Stoll (1989), Huang and Stoll (1997), and Bollen et al. (2004) to compute the inventory holding cost components. The values of the components as a percentage of the spread outside the theoretically plausible range of zero to one are excluded. LNTS is the log of the average number of trades for each stock. LTSIZE is the log of average trade size for the stock. LVOL is the log of total daily volume. LOUTSHR is the log of outstanding number of shares. SDMID is the standard deviation of midpoint. N gives the number of observations and the EXP column presents the expected sign of the relationship between each impact variable and the inventory holding component. Panel A, the sample size is different for each inventory holding premium model. In Panel B, the sample size is the same for each group.*** Statistically significant at the 0.01 level. ** Statistically significant at the 0.05 level. * Statistically significant at the 0.1 level.

In Table 5, the direction of the correlations is consistent between Panels A and B, with the differences in significance and strength of the relationships. For both the S89 and the HS models, all results are of the expected signs but not all are strong or significant. For the BSW model, all correlations are positive, counter to predictions, for both the full and subsample sets.

Focusing on the impact variables individually, the number of trades, share volume and share outstanding are negatively correlated with the estimates of inventory holding costs of S89 and HS.

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[10] We recalculate the Spearman correlation coefficient including the implausible estimates of Huang and Stoll (1997) and Stoll (1989). The Spearman correlation coefficients between variables do not change notably.

These findings are consistent with Duan and Jung-Chu (2007) and Demsetz (1968), the latter of whom argues that in an actively traded market the dealer can correct sub-optimal inventory positions with greater speed, thereby lowering the bid-ask spread. The BSW model reports positive correlations with all three variables.

Trade size is positively correlated with all three inventory holding cost components, although not statistically significant for the BSW model and weaker in all three cases for the subsample. This positive correlation is consistent with Lin, Sanger and Booth (1995) and indicates that larger trade sizes cause more order imbalances and therefore the specialist charges higher prices to balance his inventory and is consistent with expectations.

The standard deviation of the spread midpoint is positively correlated with all three inventory holding models as expected however the correlation is statistically significant only for the S89 model (both samples) and the BSW model (subsample only). The positive correlation implies that the more volatile the stock, the greater the inventory holding problem faced by the specialist. This increases the inventory holding component of the bid-ask spread which supports Ho and Stoll (1981) and is consistent with expectations. However, we cannot ignore the obvious relationship between volatility and the adverse selection component of the spread which could be affecting our results.

The correlations reported in Table 5 between the inventory holding costs and our impact variables are generally consistent with expectations for the S89 and HS models. The BSW model is positively correlated with all variables which is counter to predictions for three of the impact variables.

Overall, the results of the correlation analyses offer mixed results. Clearly the inventory holding estimates are directly and significantly related to the spread, but with varying strength. While a positive correlation is certainly expected, if the correlation is too strong, perhaps the inventory cost estimate is merely some transformation of the spread itself. Conversely, if the relationship is too weak, the inventory cost estimate may be failing to capture some portion of the cost.

As expected, the order imbalance variables are all strongly and directly correlated with each other, suggesting that all three are measuring a similar imbalance condition. A real concern, however, is the strongly negative relationship between these variables and the spread as well as with all three inventory cost estimates. If these order imbalance metrics are correctly reflecting imbalances in the market, some affect should be visible in the spread and in the inventory costs. One explanation might be that increases in the order imbalance variables reflect an increase in overall trading which would facilitate correcting imbalance conditions.

Finally, the relationships between the inventory cost estimates and our impact variables reported in Table 5 are as expected in terms of direction, again with varying levels of strength and significance, for the Stoll and Huang and Stoll models. The BSW model is positively correlated with both shares outstanding, share volume, and number of trades counter to expectations.

4.3 Regression Analysis of Inventory Holding Components and Order Imbalances

Since univariate analysis is subject to problems due to scaling and spurious correlations, we focus on ordinary least squares regression analysis with both independent and dependent variables in log-linear form.

One of the challenges in determining the performance of any estimation model is establishing valid benchmarks. Throughout the regression analysis, we use order imbalances, expressed as number of shares (OIBSH), dollars (OIBDOL), and transactions (OIBNUM), as benchmarks [(Chordia, Roll, and Subrahmanyam (2002) and Bessembinder (2002)] along with the percentage bid-ask spread (PSPREAD). The benchmark regressions use order imbalances and the spread as the dependent variable. If the inventory holding models provide any improvement over any of the order imbalance metrics or the spread as a whole, we would expect them to have more of the predicted relations.

As we are interested in cross-sectional relations, we have one inventory holding cost estimate per firm. Due to constraining the inventory holding components to values between zero and one for the S89 and HS models, we lose observations. Hence, the results of the regressions in Table 6 Panel A reflect different sample sizes for each model. The results of the regression analysis for which each model has theoretically plausible estimates of inventory holding costs (the subsample) are presented in Panel B of Table 6.

We test the following model:

$$\begin{aligned} LIHP = {} & \beta_0 + \beta_1 LOUTSHR + \beta_2 LNTS + \beta_3 LTSIZE + \beta_4 LVOL + \beta_5 LDOLVOL + \\ & \beta_6 SDMID + \ \beta_7 LPRI + \beta_8 DUMMY + e_1 \end{aligned} \tag{14}$$

where;
LIHP =ln (Inventory Holding Cost /Price) = S89, HS, BSW
or = ln(Percentage Spread/Price) = PSPREAD
or = ln(Absolute Value of Order Imbalances /Price) = OIB*aaa*
LOUTSHR=ln (number of outstanding shares)
LNTS =ln (number of average trades)
LTSIZE =ln (average trade size)
LVOL =ln(average total daily volume)
LDOLVOL=ln(average total daily dollar volume)
LSDMID =ln(standard deviation of spread midpoint)
LPRI =ln (price)
DUMMY =1 if the stock has traded options or 0 otherwise

Detailed results for all seven regressions are shown in Table 6, Panels A and B. To allow us to focus specifically on the direction and strength of the relationships determined in the regression analysis, Table 7 recaps the results in both Panels of Table 6, reporting only the sign (+/-) of the coefficient and whether or not the coefficient is statistically significant (non-significant results are in parentheses). The streamlined format of Table 7 allows us to look at all three elements of our analysis simultaneously while still considering the differing expectations for each variable set.

Our first concern is the validity of the order imbalance variables as measures of the presence of an imbalance. Looking at the four right-most columns (columns 7-10) with the heading "Presence of Imbalance," we review the relationship between our impact variables and the three order imbalance variables. Note that of the eight impact variables listed in column 1, six are expected to relate directly to an imbalance condition. The expected relationships are shown in column 10 (EXP). The availability of traded options is predicted to impact inventory holding cost but would not necessarily relate directly to the presence of an imbalance. Price is also considered inapplicable as it is a control variable related to the inventory costs. The six remaining impact variables are shares outstanding, number of trades, trade size, share volume, dollar volume and volatility. We expect all six to be positively related to an order imbalance condition.

Of these six selected variables, the order imbalance results in Panel A (full sample) produce consistently signed and significant relationships with four (shaded grey). All three order imbalance variables are negatively related to share volume, counter to expectations. In Panel B, the subsample set, the order imbalance variables produce only three consistently positive relationships, with the other three (shares outstanding, trade size and share volume) reporting mixed results. The adjusted R^2 results for all three order imbalance regressions in both panels are above 97%, indicating that our selected impact variables explain a very high percentage of the variation in the order imbalance metrics.

The columns labeled "Impact on Inventory Component" (Columns 2-6) recap the relationships between the total spread and our three model estimates with the impact variables. Note that for this set of regression results, the direction of our expected results are not the same as for the order imbalance variables. For example, while more trades (*LNTS*) would increase the order imbalance (+), an increase in number of trades is expected to facilitate correcting an imbalance, thus we expect a negative relationship between number of trades and the spread and the inventory holding cost results.

Table 6

Regressions of Inventory Holding Components and Order Imbalances on Proxy Variables of Inventory Holding Cost

	PSPREAD	S89	HS	BSW	OIBDOL	OIBNUM	OIBSH
Panel A. Different sample size for each group							
CONSTANT	0.7457***	7.1618***	3.5474***	2.8595	0.1180***	-0.2752***	0.0961
LOUTSHR	-0.1278***	-0.4446***	-0.0756	-0.1523***	-0.0131***	0.0030	-0.0240***
LNTS	-0.0000*	-0.0000	-0.0001	0.0001***	0.0000***	0.0001***	0.0000***
LTSIZE	-0.0403	0.4867***	-0.0681	0.7582****	0.1270***	0.5772***	0.3756***
LVOL	-0.0000***	-0.0000	-0.0000***	-0.0000***	-0.0000	-0.0000***	-0.0000***
LDOLVOL	-0.2935***	-0.6302***	-0.4912**	-0.1576**	0.9307***	0.8000***	0.8018***
LSDMID	0.0330***	0.0596***	-0.0357	0.0309***	0.0065***	0.0059***	0.0062***
LPRI	-0.3614***	-1.3420***	-0.1225	-1.2401***	-0.9577***	-1.7280***	-1.7496***
DUMMY	-0.0924***	-0.2200***	-0.1442**	0.0272	-0.0104***	0.0047	0.0018
Adj. R^2	0.8395	0.7444	0.1608	0.5760	0.9920	0.9773	0.9891
N	1,175	1,127	598	1,175	1,175	1,175	1,175
Panel B. The same sample size for each group							
CONSTANT	0.2030	5.1047***	3.3073***	2.9021***	0.1152	-0.0668	0.1102
LOUTSHR	-0.1375***	-0.3518***	-0.10987	-0.0371***	-0.0198**	0.0072	-0.0194**
LNTS	-0.0000	-0.0000	-0.0003***	0.0001**	0.0000***	0.0001***	0.0000***
LTSIZE	0.3234***	0.6966***	0.8778**	0.2782**	0.2018***	-0.7561***	0.2032***
LVOL	-1.3663***	-0.6716*	-1.4953**	-1.0372***	-0.2131***	0.7474***	0.7896***
LDOLVOL	0.9876***	-0.2881***	0.5720	-1.1854***	1.1077***	0.1071**	0.1050**
SDMID	0.0324***	0.0720***	0.0358	0.0188**	0.0057***	0.0049***	0.0057***
LPRI	-1.5324***	-2.5068***	-1.0492*	-0.0871***	-1.1095***	-1.0850***	-1.1069***
DUMMY	-0.0577***	-0.1989***	-0.1449**	-0.0268*	-0.0026	-0.0087**	-0.0027
Adj. R^2	0.8471	0.7605	0.1469	0.5695	0.9873	0.9793	0.9860
N	566	566	566	566	566	566	566

Notes: This table presents results for seven regressions, each with the same set of independent variables, but each with a different dependent variable. Our regression equation is:

$$LIHP = \beta_0 + \beta_1 LOUTSHR + \beta_2 LNTS + \beta_3 LTSIZE + \beta_4 LVOL + \beta_5 LDOLVOL + \beta_6 SDMID + \beta_7 LPRI + \beta_8 DUMMY + e_1$$

In three regressions, *LIHP* is the natural log of each of our three inventory holding component estimates scaled by share price. We use the models of Stoll (1989), Huang and Stoll (1997), and Bollen et al. (2004) to compute the inventory holding cost components and use the inventory holding components constrained to >0 and <1 and scaled by stock price (S89, HS and BSW). In three more regressions, *LIHP* is the natural log of the absolute value of our three order imbalance measures, OIBDOL, OIBNUM, and OIBSH, each scaled by price. OIBNUM is the absolute order imbalance in terms of number of trades, OIBDOL is the order imbalance in terms of dollar volume, and OIBSH is the absolute order imbalances in terms of number of shares. In the seventh regression, *LIHP* is the natural log of the spread scaled by the stock price (PSPREAD). *DUMMY* is the dummy variable for option listings and it is equal to 1 if the underlying stock has traded options otherwise is equal to 0. *LOUTSHR* is the log of the number of shares outstanding. *LNTS* is the log of the average number of trades for the stock. *LTSIZE* is the log of the average trade size for the stock. *LVOL* is the log of total daily average share volume. *LDOLVOL* is the log of total daily average dollar volume. *LSDMID* is the log of standard deviation of midpoint. *LPRI* is the log of the stock price. Significance is based on White-corrected t-stats. . *** Statistically significant at the 0.01 level. ** Statistically significant at the 0.05 level. * Statistically significant at the 0.1 level.

Looking first at the impact variables, we find consistent, statistically significant relationships of the predicted sign for three variables versus the spread and all three inventory holding cost estimates. The coefficient of trade size is positive consistent with Lin, Sanger and Booth (1995). The share volume coefficient is negative as predicted by Duan and Jung-Chu (2007) and the sign of the dummy variable reflecting the availability of traded options is negative as suggested by Bollen, Smith and Whaley (2004). Two additional impact variable results are consistent across the spread and two of the three inventory holding estimates but not statistically significant for the HS model. The sign of the coefficient for number of shares outstanding is negative as predicted by Duan and Jung-Chu (2007) while the coefficient of the volatility variable is positive as found by Ho and Stoll (1981). Of the remaining variables, results for both dollar volume and number of trades are mixed in terms of both sign and significance. The coefficient of price is consistently negative as expected as it is serving as a control variable.

Table 7
Regression Results showing only Sign and Significance

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Panel A. Different sample size for each group									
		Impact on Inventory Component				Presence of Imbalance			
	EXP	PSPREAD	S89	HS	BSW	OIBDOL	OIBNUM	OIBSH	EXP
CONSTANT		+	+	+	(+)	+	-	(+)	n/a
LOUTSHR	-	-	-	(-)	-	-	(+)	-	+
LNTS	-	-	(-)	(-)	+	+	+	+	+
LTSIZE	+	(-)	+	(-)	+	+	+	+	+
LVOL	-	-	(-)	-	-	(-)	-	-	+
LDOLVOL	-	-	-	-	-	+	+	+	+
LSDMID	+	+	+	(-)	+	+	+	+	+
LPRI	-	-	-	(-)	-	-	-	-	n/a
DUMMY	-	-	-	-	(+)	-	(+)	(+)	n/a
VS. EXP*		7	6*	3*	6		4*		
Adj. R^2		0.8395	0.7444	0.1608	0.5760	0.9920	0.9773	0.9891	
N		1,175	1,127	598	1,175	1,175	1,175	1,175	
Panel B. Same sample size for each group									
		Impact on Inventory Component				Presence of Imbalance			
	EXP	PSPREAD	S89	HS	BSW	OIBDOL	OIBNUM	OIBSH	EXP
CONSTANT		(+)	+	+	+	(+)	(-)	(+)	n/a
LOUTSHR	-	-	-	(-)	-	-	(+)	-	+
LNTS	-	(-)	(-)	-	+	+	+	+	+
LTSIZE	+	+	+	+	+	+	-	+	+
LVOL	-	-	-	-	-	-	+	+	+
LDOLVOL	-	+	-	(+)	-	+	+	+	+
LSDMID	+	+	+	(+)	+	+	+	+	+
LPRI	-	-	-	-	-	-	-	-	n/a
DUMMY	-	-	-	-	-	(-)	-	(-)	n/a
VS. EXP*		6*	7*	5*	7		3		
Adj. R^2		0.8471	0.7605	0.1469	0.5695	0.9873	0.9793	0.9860	
N		566	566	566	566	566	566	566	

Notes: (.) = not statistically significant , * = count excludes correctly signed but statistically insignificant results For ease of discussion and understanding, this table repeats the results from Table 6, streamlined to report only the sign of each coefficient and whether it is significant (at any level) or not. "VS. EXP" reports the number of results consistent with expectations.

Looking at the spread itself, the signs of the relationships are consistent with expectations for seven of the eight variables in the constrained sample and with six of eight in the smaller subsample (shaded). In Panel A, the direction of the result for trade size is unexpectedly negative but not statistically significant. In Panel B, number of trades remains negative but becomes statistically insignificant while the signs of trade size and dollar volume change to positive and significant. With an adjusted R^2 of about 84% in both panels, our impact variables explain a high portion of the variation in the spread.

For the three inventory holding cost estimates (columns 4-6), the signs of the relationships are consistent with more of our expectations for the S89 and BSW models, though the R^2 values for the S89 model of about 75% are much stronger than those for the BSW model (57%).

The HS model has the least number of consistent results. Though the number is improved when only the subsample is considered (Table 7 Panel B), the results are still inferior to those of the other two models. Additionally, R^2 values in the 15% to 16% range suggest that the HS model is either measuring some other element in the spread or is being driven by factors not reflected in our impact variables.

Rather than alleviating some of the confusion generated by the correlation analysis, the OLS regressions seem to have added to it. Variation in the order imbalance variables is very well explained by the impact variables suggesting that this choice of variables is appropriate for identifying inventory imbalances. Yet these same order imbalance variables are strongly, negatively correlated with both the spread and all three inventory holding cost estimates, an incongruity difficult to reconcile.

Variation in the spread is well explained by the impact variables ($R^2 \approx 0.85$), again implying that these variables are important in explaining the spread itself in the directions predicted. This is no surprise as these same variables are well-documented as strong determinants of the spread. Both the S89 and BSW models are consistent in coefficient signs with the spread regression though the ability of the impact variables to explain the Stoll 1989 model is much stronger than that of the BSW model (R^2s of about 75% versus 57% respectively). While these results for the S89 and BSW models are consistent with the spread regression, a question arises as to whether this is a positive result implying that these two models are doing a better job of isolating the inventory component or is it an indication that these two models are merely altered versions of the spread? This could well be the case especially for the Stoll model estimates. The S89 inventory costs are highly positively correlated with the volatility metric which is an acknowledged driver of both inventory and adverse selection costs.

The BSW model results in a high number of regression results consistent with predictions and with the spread results, but two of these are cause for concern. In the univariate correlations reported in Table 5, both shares outstanding and share volume are unexpectedly positive and significant which shifts to negative and significant (as predicted) in the regression. This change in direction suggests some interaction among the impact variables not captured in the Spearman correlations.

The Huang-Stoll model has slightly less of the predicted relationships with the impact variables and a weak explanatory power of about 15%. Clearly, the HS model suffers from serious drawbacks, beginning with the high number of implausible estimates. Combined with the low explanatory power of the impact variables, one suspects that the HS model is either measuring some other element of the spread or else is driven by variables unaccounted for in our impact set.

5. Conclusions

We examine the performance of three spread decomposition models specifically focusing on the inventory holding cost by comparing their component estimates to other measures of inventory holding costs. We believe that such analysis is essential if these models are to have practical use in empirical finance.

Under the assumption that trade flow imbalances are a primary driver of inventory issues, we benchmark the performance of the inventory holding cost component estimates against three measures of order imbalance expected to identify the presence of an imbalance condition as well as against the spread itself.

We analyze the performance of these models by comparing their inventory component estimates with variables selected to reflect conditions impacting the level of inventory costs. These impact variables include the number of shareholders, trading frequency, volatility, volume, and the availability of traded options.

While our analysis focuses on the ability of three spread decomposition models to estimate the inventory component of the spread, in performing this analysis we address three questions. The first issue is whether our order imbalance variables accurately measure inventory imbalance conditions. Results suggest that these benchmark metrics are measuring the same condition since the correlations are strongly positive. In addition, the regression results confirm that a high degree of the variation in the order imbalance variables (98+%) is explained by impact variables.

Our second concern addresses the relationship of the inventory holding cost estimates with both the benchmark and impact variables. Here, results are clearly mixed. In both the correlations and the regression results, the S89 and BSW models report the same number of predicted relationships with the impact variables and within one of the number of relationships between the spread and the impact variables. An additional concern is the significant negative relationship between the order imbalance variables and both the spread and all three inventory holding cost estimates. As previously noted, if the order imbalance variables do in fact identify the existence of an inventory imbalance condition then we should see a positive impact on both the spread and on the inventory holding cost estimates.

Our final and most important question is whether the model estimates improve on the spread as an indicator of inventory holding costs. Our results are generally inconclusive. The Huang and Stoll model stands out from the rest and not in a positive manner. Beginning with a very high percentage of implausible estimates, the HS model results in the least number of relationships consistent with predictions and a very low R^2 ($\approx$ 15%) in the regressions. Overall, it suggests that the Huang and Stoll model is either measuring something quite different or is being driven by factors not included in our impact variable list.

Between the remaining two models, the Stoll (1989) model results in a low number of implausible estimates (48), a high number of relationships consistent with expectations in both the correlations and regressions, and a relatively high R^2 ($\approx$ 75%). The Bollen, Smith and Whaley (2004) model results in no implausible estimates, a clear strength. This model also reports a high number of relationships consistent with expectations, however, the regression's R^2 ($\approx$ 57%) is weaker than the Stoll model. In addition, the direction (sign) of two relationships changes between the Spearman correlations and the regression results suggesting some unseen variable interaction.

With such inconclusive results, it is difficult to identify any one model as being clearly superior at isolating and estimating the inventory holding component of the spread. While the Stoll (1989) and Bollen, Smith, and Whaley (2004) models emerge as the strongest of the three studied, several unresolved issues remain. The overall relationship results for these two models are relatively consistent, yet their mean inventory holding cost estimates are quite different. The Stoll model reports an average for the constrained sample of 0.0530 while the BSW model reports an average of 0.2152 – a wide variation. The Stoll model's results are so strongly similar to those of the spread itself that it raises the concern that this model's results could be merely weak spread metrics.

Also, we cannot ignore the unexpectedly negative relationship between the order imbalance variables and the spread and between these same variables and all three inventory holding cost estimates. While the regression results confirm that our impact variables strongly capture the variation in these benchmark variables (R^2 values >97%), the inverse relationship with the spread and its components is more difficult to reconcile. One explanation might be that larger imbalances also reflect increased trading which serves to reduce both the spread and the inventory holding component. Alternately, the order imbalance variables may be capturing other elements of trading making them less reliable as benchmarks for our study.

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