Contingent Claim Valuation of Express Certificates

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In this paper we introduce a new financial product named Express Certificates and we provide detailed descriptions of the product specifications. We show that the payoff of an Express Certificate can be duplicated by the combination of a zero coupon bond, a cash-or-nothing call option on the index and a put option on the index. We develop a pricing formula to price the certificates. Finally, we apply the pricing model for Express Certificates to a certificate issued by Bayerische Hypo- und Vereinsbank AG to examine how well the model fits empirical data. The results are in line with previous studies pricing other structured products.
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1. Introduction

In the financial engineering process – i.e. the creation of new securities combining fixed income securities, equities, and derivative securities – investment and commercial banks have constantly increased the *complexity* of securities. As a result, regulators have expressed concerns that some of these exotic products may be too complicated for individual investors at the retail level to understand and propose limiting the participation only to qualified investors – i.e. sophisticated investors (Ricks, 1988; Lyon, 2005; NASD, 2005; Laise, 2006; Maxey, 2006; Simmons, 2006; Isakov, 2007).

In this paper, we study a new financial product known as "*Express Certificates*" (to be referred to as EC henceforth), one of the new equity-linked "structured products" issued by major banks in Europe. The rate of return on the investment in the certificates is contingent upon the performance of a *pre-specified underlying equity* or *equity index* over a *pre-specified period* (known as *term to maturity*). As long as the underlying asset price does not close on maturity date below a predetermined level referred to as the *knock-in level,* [1] the investors of the certificates will receive a guaranteed minimum redemption amount at maturity[2]. If, however, the price of the underlying asset closes on maturity date below the knock-in level, the investor is fully exposed to the decline in the underlying asset. In calculating the return on the underlying asset, the certificate issuers will use only the change in the asset price; the cash dividends paid during the period are not included. In other words, investors in the ECs do not receive cash dividends even though the underlying assets pay dividends during the term to maturity. One attractive feature of this certificate is that the premium in the guaranteed minimum redemption is tax free in Germany. A 5% tax free premium in a guaranteed minimum redemption amount of 105% of nominal value would have a comparable taxable 9.61% return (i.e. using a combined tax rate of income tax and social security taxes of 48%).

The purpose of the paper is to provide an in-depth economic analysis for the ECs to explore how the principles of financial engineering are applied to the creation of such newly structured products. We also develop a pricing model for the certificates by using option pricing formulas. In addition, we present an example of an EC issued on October 29, 2004 by Bayerische Hypo- und

[1] Usually the knock-in level is set up as a percentage of the *initial price* (e.g. 75% of the initial price). A certificate with a knock-in level of, for example, 75% of the initial price, is also referred to as having a 25% *downside protection*.
[2] The guaranteed minimum redemption amount may be the same as or higher than the par amount of the certificates.

Vereinsbank AG (to be referred to as *HVB Bank* henceforth), a well-recognized large bank in Europe. In this example, we practice the pricing of the certificate by calculating the price of a portfolio with a payoff similar to the payoff of the certificate.

The rest of the paper is organized as follows: The design of the certificates is introduced in Section 2. The pricing model of the certificates is developed in Section 3. We present an example of EC in Section 4 and empirically calculate the profit in the primary market for issuing the certificate using the option pricing model developed in Section 3. We conclude the paper in Section 5.

2. Description of the Product

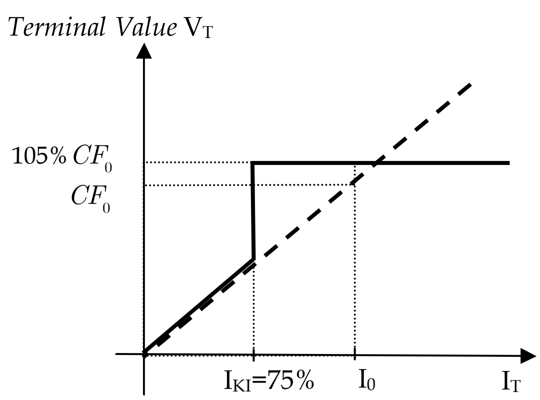
The rate of return of a certificate is contingent upon the price performance of its underlying asset over its term to maturity. The *beginning date* for calculating the gain (or loss) of the underlying asset is known as the *fixing date* (or *pricing date*) and the *ending date* of the period is known as the *expiration date.* The price of the underlying asset on the *fixing date* is referred to as the *reference price* (or *exercise price,* or *strike price*), and the price of the underlying asset on the *expiration date* is referred to as the *valuation price*[3].

If we denote I_0 as the underlying asset price on the fixing date, I_T as the *valuation price*, p as the *premium* (*discount*) received in the guaranteed minimum redemption as a percentage of the nominal value of the certificate, k as the *knock-in level* as a percentage of the nominal value of the certificate, then for an initial investment of CF_0 in an *express* certificate, the total value that an investor will receive on the *expiration date* (known as the *redemption value* or *settlement amount*), V_T, is equal to:

$$V_T = \frac{CF_0}{I_0}\begin{cases} I_0 \ (1+p) & if\ I_T \geq k\ I_0 \\ I_T & if\ I_T < k\ I_0 \end{cases} \tag{1}$$

Alternatively, the relationship between the terminal value of an *express* certificate and the terminal value of the underlying asset based on the change in the underlying asset price (without taking into account dividends) with a knock-in level of 75% of the exercise price (also known as a capital protection of 25%) can be represented in Figure I.

Figure I



Notes: The terminal value of an investment of CF_0 in an Express Certificate as a function of terminal index I_T, with a downside protection of 25%. The solid line represents the terminal value of the certificate on maturity day T, as a function of the terminal value of the underlying index. The dotted line represents the terminal value of the underlying index.

The slope for the value of the underlying asset in Figure I is, of course, one. The slope for the value of the certificate, when the price of the underlying asset goes *down*, is also equal to one.

[3] In the example presented in Section 4 the *exercise price* and the *valuation price* are the *closing* prices on the *fixing date* and the *expiration date* respectively.

3. The Pricing of Express Certificates

The terminal value from Equation (1), V_T, for an initial investment of CF_0 in an express certificate with an exercise price of I_0, a knock-in level of $k*I_0$, and term to maturity T, can be expressed mathematically as:

$$V_T = CF_0 \left(1+\mathrm{p}\right) \; \mathrm{I}_{[kI_0,\infty)}\left(\mathrm{I}_T\right) + \frac{CF_0}{I_0} I_T \; \mathrm{I}_{[0,kI_0)}\left(\mathrm{I}_T\right) \tag{2}$$

where $\mathrm{I}_{[kI_0,\infty)}\left(\mathrm{I}_T\right)$ is an indicator function that its value is equal to one when the underlying asset price at maturity is equal or greater than $k*I_0$ and zero otherwise. $\mathrm{I}_{[0,kI_0)}\left(\mathrm{I}_T\right)$ is an indicator function that its value is equal to one when the underlying asset price at maturity is smaller than $k*I_0$ and zero otherwise.

$$V_T = kCF_0 \, I_{[kI_0,\infty)}\left(I_T\right) + \left(1-k+p\right)CF_0 \, I_{[kI_0,\infty)}\left(I_T\right) + \frac{CF_0}{I_0} I_T I_{[0,kI_0)}\left(I_T\right)$$

$$V_T = kCF_0 \; \mathrm{I}_{[kI_0,\infty)}\left(I_T\right) + \left(1-k+p\right)CF_0 \; \mathrm{I}_{[kI_0,\infty)}\left(I_T\right) + kCF_0 \; \mathrm{I}_{[0,kI_0)}\left(\mathrm{I}_T\right) - \frac{CF_0}{I_0}\max\left(kI_0 - I_T, 0\right)$$

$$V_T = kCF_0 + \left(1-k+p\right)CF_0 \; \mathrm{I}_{[kI_0,\infty)}\left(I_T\right) - \frac{CF_0}{I_0}\max\left(kI_0 - I_T, 0\right) \tag{3}$$

The $\max\left(kI_0 - I_T, 0\right)$ in Equation (3) is the payoff for a put option with an exercise price of $k*I_0$. The payoff of one EC, presented in Table 1, is exactly the same as the payoff for holding the following three positions:

1. A long position in one zero coupon bond with face value equal to k* CF_0 and maturity date same as the maturity date of the certificate;
2. A long position in cash-or-nothing call options with exercise price of $k*I_0$, term to expiration of T (which is the term to maturity of the certificate), and number of options of (1-k+p)* CF_0.
3. A short position in put options with exercise price of $k*I_0$, term to expiration of T (which is the term to maturity of the certificate), and number of options of CF_0/I_0.

Table 1
Payoffs at expiration for the portfolio identical to the certificate

Security	Current Value	$I_T < k* I_0$	$I_T > k*I_0$
Long Zero Coupon Bond	CF_0*k*e^{-rT}	CF_0*k	CF_0*k
Long Cash-or-Nothing Call	$CF_0*(1-k+p)*C$	0	$CF_0*(1-k+p)$
Short Puts	CF_0/ I_0*P	$- CF_0*k+ I_T* CF_0/ I_0$	0
Express Certificate	$CF_0 (k*e^{-rT}+(1-k+p)*C-P/ I_0)$	$I_T* CF_0/ I_0$	$CF_0*(1+p)$

Since the payoff of a certificates is the same as the combined payoffs of the above three positions, we can calculate the fair value of the certificates based on the value of the three positions. Any selling price of the certificates above the value of the above three positions is the gain to the certificate issuer.

The value of *Position 1* is the price of a zero coupon bond with a face value k* CF_0 and maturity date T. So it has a value of $k^*CF_0e^{-r\ T}$. The value of *Position 2* is the value of (1-k+p)* CF_0 shares of cash-or-nothing call options with exercise price X (≡ k*I_0) and each option having the value C:

$$C = e^{-rT}N(d_2) \tag{4}$$

where r is the risk-free rate of interest, T is the term to maturity of the certificate and

$$d_1 = \left(\ln\left(\frac{1}{k}\right) + \left(r - q + \frac{1}{2}\sigma^2\right)T\right) \Big/ \sigma\sqrt{T} \tag{5}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{6}$$

where σ is the standard deviation of the underlying asset returns and q is the dividend yield of the underlying asset. The value of *Position 3* is the value of 100/I_0 shares of put options with each option having the value P:

$$P = Xe^{-rT}N(-d_2) - I_0e^{-qT}N(-d_1) \tag{7}$$

where r is the risk-free rate of interest, q is the dividend yield of the underlying asset, T is the term to maturity of the certificate, d_1 and d_2 are defined in Equations (5) and (6) respectively, and X (≡ k*I_0) is the exercise price. Therefore, the total cost, TC, for each certificate is

$$TC = kCF_0e^{-rT} + \left(1 - k + p\right)\ CF_0C - \frac{CF_0}{I_0}P \tag{8}$$

If we denote B_0 as the issue price of the certificate, any selling price above the fair value is the gain to the certificate issuer. And the profit function for the issuer of certificates is

$$\Pi = B_0 - TC \tag{9}$$

The profitability is measured by the profit, ∏, as a percentage of the total issuing cost, TC:

$$\text{Profitability} = \frac{B_0 - TC}{TC} * 100\% \tag{10}$$

4. Empirical Test

In this section, we empirically examine an EC issued by HVB Bank on October 29, 2004 using the Dow Jones Euro STOXX 50 as the underlying asset. The EC is the HVB Express Certificate (ISIN DE000HV0AZU0), and the major characteristics of the certificate are listed in Appendix 1 of the paper.

Based on the information in Appendix 1, the certificate has a guaranteed minimum redemption 105% of the nominal value of the certificate (a 5% premium), and a 25% downside protection on the negative returns of the underlying asset. The fixing date HVB set for the certificate was October 26, 2004, when the Dow Jones Euro STOXX 50 Index value was 2,739.37, and the issue price of the certificate was €100 plus a sales charge per €100 nominal value. The sales charge in similar securities is in the range of 0.5% and 2%. The expiration date (i.e. the date on which the closing price of the underlying asset will be used as the valuation price) was set on December 16, 2005, 1.1379 years later. Therefore, the payoff to the investor of on maturity date, T, is:

$$V_T = €75 + €30I_{[2,054.53,\infty)}\left(I_T\right) - \frac{€100}{2,739.37}\max\left(2,054.53 - I_T, 0\right) \tag{11}$$

The cost of the payoff of €75 in Equation (11) is €75 $e^{-r\ 1.1370}$, the cost of the payoff €30*$I_{[2,739.37,\infty)}$ (I_T) is 30 cash-or-nothing call options with exercise price of 2,054.53, and the cost of the payoff €100/2,739.37 max (2,054.53-I_T,0) is €100/2,739.37 put options with an exercise price of 2,054.53. The call premium can be calculated from the following equation:

$$C = e^{-r1.14}N(d_2) \tag{12}$$

where

$$d_1 = \left(0.2877 + \left(r - q + \frac{1}{2}\sigma^2\right) * 1.1370\right) \Big/ \sigma\sqrt{1.1370} \tag{13}$$

$$d_2 = d_1 - \sigma\sqrt{1.1370} \tag{14}$$

The put premium can be calculated from the following equation:

$$P = 2{,}054.53e^{-r\ 1.1370} N(\text{-}d_2) - 2{,}729.37e^{-q\ 1.1370} N(\text{-}d_1) \tag{15}$$

where can d_1 and d_2 can be calculated using Equations (13) and (14) respectively. The total cost of the certificate, TC, is

$$TC = €75\ e^{-r1.1370} + 30C - \frac{€100}{2{,}739.37}P \tag{16}$$

where C is the call premium calculated in Equation (12) and P is the put premium calculated in Equation (15). The issuer sells the certificate for €100, therefore the profit for issuing the certificate, π, is equal to

$$\Pi = €100\text{-}\left(€75\ e^{-r\ 1.1370} + €30C - \frac{€100}{2{,}739.37}P\right) \tag{17}$$

In order to calculate the issuer's profit, we need the following data for the certificate: 1) the price of the underlying asset, I_0, 2) the cash dividends to be paid by the underlying assets and the ex-dividend dates so we can calculate the dividend yield, q[4], 3) the risk-free rate of interest, r, and 4) the volatility of the underlying asset, σ.

The prices and dividends of the underlying asset are obtained from Bloomberg; the risk-free rate of interest is the yield of government bonds (alternatively, swap rates) of which the terms to maturity match those of the certificate[5]. The volatilities (σ) of the underlying assets are the implied volatility obtained from Bloomberg based on the *call* and *put* options of the underlying asset[6].

The risk-free rate of interest, r, on October 26, 2004, the issue date of the certificate, based on the Euro swap rates is 2.36%. The dividend yield, q, on the Dow Jones Euro STOXX 50 Index is 0.76%. The Dow Jones Euro STOXX 50 Index value on the issue date of the certificate, I_0, is 2,739.37. The implied volatility of the Dow Jones Euro STOXX 50 Index based on the index call (put) potions was 18.04% (16.66%) on the issue day. Therefore, the d_1 and d_2 in Equation (13) and (14) respectively for the call option are,

$$d_1 = 1.6863$$

$$d_2 = 1.4939$$

$$N(d_2) = N(1.4939) = 0.9324 \tag{18}$$

Substitute Equation (18) into Equation (12) and the total cost of the cash-or-nothing call option is

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[4] Equations (12) and (15) are based on continuous dividend yield. Since the dividends from the underlying security are discrete, we use the following approach to calculate the *equivalent* continuous dividend yield for underlying security that pays discrete dividends. For an underlying asset which is an index with a price I_0 at t=0 (the issue date) and which pays n dividends during a time period T with cash dividend D_i being paid at time t_i, the equivalent dividend yield q will be such that

$$I_0 - \sum_{i=1}^{n} D_i e^{-r\,t_i} = I_0 e^{-q\,T}, \qquad q = -\frac{\ln\left[1 - \frac{\sum_{i=1}^{n} D_i e^{-r\,t_i}}{I_0}\right]}{T}$$

[5] If we cannot find a government bond that matches the term of maturity for a particular certificate, we use the linear interpolation of the yields from two government bonds that have the closest maturity dates surrounding that of the certificate.

[6] The implied volatility calculated by the Bloomberg System is the weighted average of the implied volatilities for the three call (put) options that have the closest at-the-money strike prices. The weights assigned to each implied volatility are linearly proportional to the "degree of near-the-moneyness" (i.e. the difference between the underlying asset price and the strike price) with the options which are closer-to-the-money receive more weight.

$$C = e^{-0.0236*1.1370}0.9324 = €0.9077 \tag{19}$$

The d_1 and d_2 in Equation (13) and (14) respectively for the put option are,

$$d_1 = 1.8107$$

$$d_2 = 1.6330$$

$$N(-d_1) = N(-1.8107) = 0.0351 \tag{20}$$

$$N(-d_2) = N(-1.6330) = 0.0512 \tag{21}$$

Substitute Equation (20), (21) into Equation (15) and the total cost of the put option is

$$P = 2{,}054.53e^{-0.0236*1.1370}0.0512 - 2{,}729.37e^{-0.0076*\ 1.1370}0.0351 = €7.1568 \tag{22}$$

Substitute Equations (19), (22) into Equation (16), and the total cost of issuing the EC, TC, is

$$TC = €75\ e^{-0.0236*1.1370} + 30*€0.9077 - \frac{€100}{2{,}739.37}€7.1568 = €99.98 \tag{23}$$

The profit for issuing each EC, π, is

$$\Pi = €100\text{-}€99.98 + \text{ Sales Charge} \tag{24}$$

So the profit for issuing each EC with a par value of € 100 is in the range of the sales charge (i.e. between 0.5% and 2% in similar securities). A profit in the range of 0.5%-2% for an approximately 14-month period translates into an annual rate of return in the range of 0.43%-1.71%, in line with the HVB Bank's reported return on assets of 0.80% (HVB Bank's 2004 Consolidated Annual Report). The *return on assets* range of 0.43%-1.71% calculated from the pricing model in the paper can also be translated into a *return on equity* range of 8.14%-32.36% using the HVB Bank's reported financial leverage (HVB Bank's 2004 Consolidated Annual Report). The previous return on equity range is also in line with by HVB Bank's reported return on common stockholder's equity, which is 17.9%. The remarkable consistency between the empirical results calculated from the pricing model developed in the paper and the reported financial data in HVB Bank's Annual Report suggests the model developed in the paper is sound and robust.

Moreover, the result in the paper provides additional evidence of the profitability of structured products in the primary market. Several studies have reported that structured products have been overpriced, 2%-7% on average, in the primary market based on theoretical pricing models (Chance and Broughton, 1988; Chen and Kensinger, 1990; Chen and Sears, 1990; Baubonis et al., 1993; Burth et al., 2001; Wilkens et al., 2003; Grünbichler and Wohlwend, 2005; Stoimenov and Wilkens, 2005; Benet et al., 2006; Hernandez et al., 2008; Hernandez et al., 2010) for various types of structured products.

However, some caveats apply to these results. New instruments, particularly over-the-counter ones, do not have deep markets and illiquidity can be a source of premium (Amihud, 2002; Longstaff et al., 2005; Chen et al., 2007). Illiquidity can also lead to limitations on trades which are needed to unwind abnormal profits or needed to hedge the exact same term to maturity and exercise price of the certificate (Cochrane, 2002; Ofek and Richarson, 2003; Hong et al., 2006). The profit calculated at issuance is gross profit before any design or marketing cost. Finally, even though the certificate is unsecured obligation, we assumed the counterpart default risk is negligible since the issuer is a high quality bank. Considering the previous caveats, however, the magnitude of the profit is remarkable.

5. Conclusion

In this paper we introduce a newly structured product known as ECs and we provide detailed descriptions of the product specifications. We further develop pricing model for the certificates. Finally, we apply the pricing model for ECs to a certificate issued by HVB Bank to examine how well the model fits empirical data. We find that issuance of the certificate is profitable for the issuer. The result is in line with previous studies pricing other structured products.

The study provides insights into the design, the payoff, the pricing and the profitability of the newly designed financial product. The methodology and approach used in this paper can be easily extended to the analysis of other structured products.

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Appendix 1: Example of an Express Certificate

The Express Certificate in Appendix 1 was issued by investment bank HVB using the Dow Jones Euro STOXX 50 as the *underlying asset*. The *fixing date* HVB set for the certificate was October 26, 2004 and the *issue price* of the certificate was €100. The *expiration date* (i.e. the date on which the closing price of the underlying asset will be used as the *valuation price*) was set on December 16, 2005.

HVB CORPORATES & MARKETS

HVB EXPRESS CERTIFICATE - Dow Jones Euro STOXX 50

Issuer	Bayerische Hypo- und Vereinsbank AG
Index	Dow Jones Euro STOXX 50
Type	Certificate
Subscription Period	7 October – 26 October 2004
Settlement Date	29 October 2004
Maturity Date	23 December 2005
Issue Price	€100 per certificate
Denomination	€ 100
Repayment	The redemption amount is calculated as: If $I_{final} \geq 0.75\ I_{initial}$ the repayment is €105 per certificate If $I_{final} < 0.75\ I_{initial}$ the repayment is calculated as: $€100 * I_{final} / I_{initial}$ $I_{initial}$ is the official closing price of the Dow Jones Euro STOXX 50 (Price) Index on October 26, 2004 I_{final} is the official closing price of the Dow Jones Euro STOXX 50 (Price) Index on October 16, 2005
Issue Date	29 October 2004
Listing	Open Market - Frankfurt Stock Exchange (Smart Trading) Stuggart (EUWAX)
Smallest Unit	1 certificate
WKN	HV0AZU
ISIN Code	DE 000 HV0 AZU 0