# Accounting for Skewness in Performance Evaluation of Brazilian Mutual Funds

Aquiles Farias [a], José Ornelas [*b], Antônio Silva Júnior [a,b]

[a] Central Bank of Brazil, Brazil
[b] The Federal University of Bahia, Brazil

The Sharpe Ratio is probably the most widely known and used performance measure for mutual fund evaluation. However, it is based on the mean-variance theory and thus it is valid either for Normal returns or for quadratic utility functions. It does not take into account skewness in the distribution of returns. If we consider investors with aversion for negative skewness, it is beneficial to have a measure that goes beyond mean-variance. Koekebakker and Zakamouline (2009) propose a measure called ASSR (Adjusted for Skewness Sharpe Ratio) that generalizes the Sharpe ratio, accounting also for skewness. However, the ASSR may result in imaginary numbers under certain conditions. In fact, in our sample many funds obtained imaginary numbers for the ASSR. Thus, we propose a new measure that does not have to deal with imaginary numbers, but maintains the major features of the original measure of Koekebakker and Zakamouline. We used the new measure to rank Brazilian Fixed Income and Multimarkets funds. The results showed a very low ranking correlation between the new measure and the Sharpe Ratio, suggesting that skewness is an important consideration when analyzing Brazilian mutual funds.

*JEL classification*: G11
*Key words*: Sharpe Ratio, Skewness, Performance Evaluation.

## 1. Introduction

Numerous performance measures have been proposed in the literature to assess active fund managers. The Sharpe Ratio is probably the most widely known and used. But this measure of performance evaluation proves adequate only when either the investor believes that risk can be suitably measured by standard deviation, or when returns display nice symmetric distributions like the Normal. However, many Brazilian mutual funds have distributions with non-Normal shapes.

The literature shows that distribution of returns on assets and portfolios may be non-Normal, usually with fat tails and negative skewness. We could identify two reasons for this non-normality. The first is that the individual assets available themselves have non-Normal distributions. This is the case of the Brazilian market. Thus, when we add non-normal assets in the portfolio, the resulting return will also be non-Normal. The other reason is the use of derivatives, which are used to change the portfolio leverage or to add negative skewness. These strategies are often criticized as a "manipulation" of the Sharpe Ratio. The work of Goetzmann et al. (2007) analyzes several manipulative strategies of traditional measures such as the Sharpe Ratio.

Several authors have proposed measures that go beyond the mean-variance world. This is the case of Hodges (1998), Keating and Shadwick (2002), Goetzmann et al. (2007) and Koekebakker and Zakamouline (2009). The latter propose a measure that generalizes the Sharpe Ratio to account for skewness. Assuming that investors are not neutral to skewness, the measure they apply penalizes returns with negative skewness.

---

The views expressed in this work are those of the authors and do not reflect those of the Banco Central do Brasil or its members.

* Corresponding author: jrenato.ornelas@bcb.gov.br

However, they make a derivation that leads to a performance evaluation index that might result in imaginary numbers under certain conditions. In our work, we discuss their derivation and propose a new measure. We argue that an appropriate measure for ranking funds would be based on the weight allocated to the risky asset derived by their model. This new measure is more appropriate since it does not have to deal with imaginary numbers, but retains the main features of the original measure of Koekebakker and Zakamouline. Then we empirically evaluated the effects on the rankings of the Brazilian Fixed-Income- and Multimarkets funds when we used this new measure and the traditional Sharpe Ratio.

The fund industry in Brazil has grown vastly in recent years (Alves Junior, 2003) due to the growth of institutional investors like pension funds, and the changes in the regulatory framework, and economic development, among other factors. The literature on performance of funds in Brazil concentrates mostly on Equity Funds and there are few papers about Multimarkets and Fixed-Income funds. Therefore, we expect also to contribute to the debate on performance measures in the Brazilian mutual fund industry, using a measure that goes beyond mean-variance. The ultimate choice of performance measure will depend on the preferences of Brazilian Fixed-Income and Multimarket Funds investors. Almeida (2004) evaluated investors' preferences in Brazilian funds using panel data and concluded that returns are negatively correlated to skewness. Hence, there is a premium for funds with negative skewness. Funds with negative skewness have a higher downside risk and seek a higher return. Based on this finding, we can say that Brazilians are not neutral to skewness of returns distribution, and thus measures of performance that consider not only mean and variance but also other characteristics of distribution should be studied and utilized. A performance evaluation measure that treats standard deviation as the only risk measure does not get investors' preferences on a fair basis.

Section two of this paper presents a brief literature review. Section three discusses the work of Koekebakker and Zakamouline (2009) and proposes a new index that takes skewness into account in a more appropriate way. In the fourth section we present a summary of returns' data. In section five we show results of this new performance evaluation index when applied to Brazilian mutual fund data, and compare them with results of the Sharpe Ratio. Section 6 concludes the paper.

## 2. Literature Review

Silva Junior (2004) analyzes the investment process in several steps including strategic asset allocation in benchmark selection, active management, guidelines, and performance reports. Performance reports are an important task in the investment process, since they help in acquiring a better understanding of the risk-return relationship. They help asset allocation risk budgeting, strategies, managers' evaluations etc. Silva Junior (2004) divides performance reports into three complementary parts. The first is the return calculation report. Despite the fact that several measures could be used, returns calculations generally follow CFA (Chartered Financial Analysts) institute standards using time-weighted returns. Another part of performance reports is performance attribution, where we can identify sources of returns and we may attribute each source to the strategies managers have used. Finally, we have performance evaluation that is related to return adjustment based on the risks incurred by managers.

In fact, performance evaluation has many roles in the investment process. It is intrinsically associated with asset allocation because the strategic asset allocation must reflect investors' risk preferences. Risk budgeting also needs a thorough understanding of performance evaluation. Several managers view their risk budget process as maximization of a performance evaluation measure, as we can see in Dinking at al. (2007). Performance evaluation numbers are also used for selection and assessment of external portfolio managers. Géhin (2004) shows some factors that may influence performance evaluation steps like data quality, survivorship bias, instant history bias, sizes of funds, age of funds, market factors, probability distribution of returns etc. In fact, rankings also depend on the kind of measure we use.

Performance evaluation must be risk-adjusted, and traditional measures are based on a mean-variance framework as in the Sharpe Ratio, Jensen's Alpha, Treynor Ratio etc. Some variations of these approaches are used as we can see in Elton, Gruber and Blake (2003) where they use a multi-index performance measure to evaluate incentive fees for mutual funds. Lately, there is a growing body of literature on performance evaluation that tries to take into account higher moments of distribution and not merely mean and variance. We see two reasons for the emergence of these measures: first, there is a new paradigm of investors' perception of risk that goes beyond simply the variance; and second, many asset return distributions are actually non-Normal distributions. The first reason is related to the increasing use over the last 15 years of a number of risk measures that focus on the left tail of the distribution of returns, such as Value at Risk (VaR) and Expected Shortfall.

Many research studies replace standard deviation in the Sharpe ratio by risk measures that focus on the left tail of distributions. Sortino (1991) replaces the standard deviation by the downside deviation. Dowd (2000) uses the Value-at-Risk (VaR) measure instead of standard deviation. Stutzer (2000) proposes the Stutzer index, which is based on the assumption that investors want to minimize the probability of underperforming against a specific benchmark. Keating and Shadwick (2002) introduced the Omega Ratio which is defined as the ratio of the gain relative to a given threshold to the loss with respect to the same threshold. Grava and Siqueira (2003) evaluate Brazilian mutual funds using a volatility price, given by the coefficient of a GARCH-M model.

Returns of portfolios and funds are actually not normally distributed. Hedge funds are an example of odd-shaped distributions that deviate substantially from normality (see, for example Malkiel and Saha, 2005). Nevertheless, the Sharpe Ratio and similar measures are commonly used to evaluate and rank hedge funds. Cerny (2003) proposes a generalized Sharpe Ratio that according to him provides a consistent ranking of investment opportunities even when asset returns are highly non-Normal. Distribution of returns of a fund may be non-Normal simply because of the fund's holdings contain assets with non-Normal properties. However, several papers have argued that hedge fund managers use strategies that try to manipulate Sharpe-like performance measures. Goetzmann et al. (2007) describe three general strategies for manipulating a performance measure: the first is manipulation of the underlying distribution in order to influence the measure; the second is dynamic manipulation that introduces time variation into the return distribution to influence measures that are assumed stationary; the last strategy is a kind of dynamic manipulation that focuses on inducing estimation errors.

In fact, major papers in recent performance measurement literature try to capture the skewness of returns distributions. Such literature argues that investors are not neutral to skewness and hence there is a premium associated with skewness. Many approaches are derived from utility functions that take into account investors' preferences to higher moments of returns distributions. A very interesting example is the work of Koekebakker and Zakamouline (2009). Based on the Generalized Sharpe Ratio of Hodges (1998), they propose two performance measures, one a general performance measure taking into account only skewness and other taking into account both Skewness and Kurtosis. The first takes into account skewness preferences in investment decisions and derives the Adjusted for Skewness Sharpe Ratio (ASSR). Despite the very comprehensive nature of this measure it has a major drawback: it might generate complex numbers in some cases rendering comparison of different measures not possible.

The Adjusted for Skewness and Kurtosis Sharpe Ratio (ASKSR) proposed by Koekebakker and Zakamouline (2009) is a measure that takes into account both Skewness and Kurtosis. However, the closed-form formula of this measure has to be derived for a specific distribution. They give an example using a Normal Inverse Gaussian (NIG). Thus, given the estimated parameters of the NIG, the ASKSR can be calculated using a formula. The problem with this approach is the estimation procedure might encounter problems of convergence and high sensibility to the choices of initial parameters, especially when the amount of data available is small.

Goetzmann et al. (2007) present numerical simulations of manipulation strategies for several performance measures like Sharpe Ratio, Jensen's Alpha, Treynor Ratio, Appraisal Ratio, Sortino and Van der Meer Ratio (1991), Sortino, Van der Meer and Plantinga (1999) ratio, and the timing measures of Henriksson and Merton (1981), and of Treynor and Mazuy (1966). Manipulations generate portfolio scores that are statistically and economically superior even though all of the simulated trades can be carried out by an uninformed investor. Goetzmann et al. (2007) suggest a manipulation-proof performance measure (MPPM) that has four properties: (i) produce a single value score with which to rank each subject; (ii) value of the score should not depend on portfolio size; (iii) while an uninformed investor cannot expect to enhance his estimated score by deviating from the benchmark, informed investors should be able to produce higher scoring portfolios using arbitrage; and (iv) the measure should be consistent with standard financial market equilibrium conditions.

Despite the wide range of measures available, Eling and Schuhmacher (2007) analyze and compare 13 different performance measures for a set of hedge fund returns and conclude that all these performance measures produce similar rankings. They use the following measures: Sharpe Ratio, Treynor, Jensen's Alfa, Omega Ratio, Sortino Ratio, Kappa 3, the upside potential ratio, the Calmer Ratio, the Sterling Ratio, the Burke Ratio, the excess return on Value at Risk, the conditional Sharpe Ratio, and the modified Sharpe Ratio. However, it is must be mentioned that recent measures like the MPPM and ASSR were not considered in this study. In fact, the authors used measures that were very similar nature and form, i.e. expected return divided by a risk measure.

Berk and Green (2004) discuss an interesting puzzle in the performance evaluation literature: the evidence of lack of consistent performance would imply that superior performance is attributable to luck rather than to the differential abilities of managers. But they argue that this implication would be distressful from an economic point of view since there are rewards for superior past performance and investors devote considerable resources to evaluation of past performance of managers. According to the author's model there is competitive provision of capital by investors to mutual funds, and there is differential ability to generate high average returns across managers, but decreasing returns to scale in deploying these abilities and there is learning about managerial ability from past returns. Some authors like Oliveira (2005) discovered that in a quarterly horizon there is performance persistence in the Brazilian Multimarket Fund Industry. The issue of ranking funds and persistence is a very important one. Lynch and Musto (2003) state that the literature documents a convex relationship between past returns and fund flows of mutual funds. Many authors argue that it is difficult to assess the estimation error associated with each measure. The work of Eid Jr. and Rochman (2005) evaluates some performance indexes based on a bootstrap procedure to assess the estimation risk. Martins (2006) also uses a bootstrap technique to assess estimation error in performance evaluation of mutual funds.

In fact, there is no consensus on performance evaluation approaches for ranking funds. Each measure produces different results that should be carefully interpreted by investors. It is not prudent for one to rely on a single measure to decide in what fund to invest one's money. Therefore, it is necessary to understand performance measurement theory, strengths and weaknesses of indexes, and investors' preferences regarding risks and returns.

## 3. A Performance Measure

Koekebakker and Zakamouline (2009) consider an investor who wants to allocate his wealth between a risk-free and a risky asset. According to their model, returns follow a stochastic process as follows:

$$x = \mu \Delta t + \sigma \sqrt{\Delta t}\,\varepsilon \qquad (1)$$

where $\mu$ and $\sigma$ are mean and volatility of the risky asset returns per unit of time $\Delta t$, and ε is a stochastic error term.

Risk-free asset returns are modeled as:

$$r_f = r\Delta t \tag{2}$$

where $r$ is the risk-free interest rate per unit of time.

Investors have a wealth of $\omega$ and invest $a$ in the risky asset and the remaining in the risk-free asset. After $\Delta t$, investors' wealth is:

$$\tilde{\omega} = a\left(x - r_f\right) + \omega\left(1 + r_f\right) \tag{3a}$$

The investors' expected utility **E** is a function of the investors' wealth, returns, and the amount invested in risky assets and in the risk-free asset:

$$E\left[U\left(\tilde{\ }\quad J\left(a\left(x - r_f\right) + w\left(1 + r_f\right)\right)\right)\right] \tag{3b}$$

Koekebakker and Zakamouline (2009) solve the problem of utility maximization with an application of a Taylor series expansion for $U\left(\tilde{\omega}\right)$ around $\omega\left(1 + r_f\right)$. They truncate the Taylor expansion in a way that takes into account the moments up to the skewness of the return distribution and disregards all the higher moments of the distribution. Thus, the solution for utility maximization gives the amount that should be invested in the risky-asset ( $a$ ) as:

$$a = \frac{SR}{\gamma\sigma\sqrt{\Delta t}}\left(1 + \frac{bS}{2}SR\right) \tag{3}$$

where $SR$ is the Sharpe ratio, $S$ is the skewness of probability distribution of returns, $\gamma$ is the Arrow-Pratt measure of absolute risk aversion, and $b$ is the investor's preferences to skewness parameter.

Since we have the solution for $a$, Koekebakker and Zakamouline (2009) show that it is possible to express the (approximate) maximum expected utility in terms of the Sharpe ratio adjusted for skewness preferences:

$$E\left[U\left(\tilde{\omega}\right)\right] \approx U\left(\omega_r\right) + \frac{U'\left(\omega_r\right)}{\gamma}\frac{1}{2}SR^2\left(1 + \frac{bS}{3}SR\right) \tag{4}$$

According to Koekebakker and Zakamouline (2009), equation (6) suggests that for any investor the higher the value of

$$SR^2\left(1 + \frac{bS}{3}SR\right) \tag{5}$$

the higher the expected utility. They argue that if the risky asset represents a portfolio of several assets and the investor's problem is to choose an optimal portfolio mix, the investor's alternative objective is to maximize the value given by (6). Therefore, Koekebakker and Zakamouline (2009) proposed the Adjusted for Skewness Sharpe ratio:

$$ASSR = SR\sqrt{1 + \frac{bS}{3}SR} \tag{6}$$

It is must be mentioned that equation (5) represents the utility function for a combination of the risky asset with the risk-free asset. If the risky asset has a negative excess expected return, the solution provided by (4) gives a negative allocation, i.e., to short-sell the risky asset. For instance, a fund with a high negative Sharpe ratio and negative skewness may improve the utility function in equation (5) because the model will short-sell this fund, and obviously it would not reflect the fund manager's superior ability. The utility of risky assets with negative excess expected return risky assets arises from the fact that one could short sell it and reinvest in the risk-free asset. This would make sense for ETFs (Exchange-Traded Funds), indexes with future contracts, or any asset with a liquid repo market.

Such a situation also occurs with the Sharpe Ratio. As pointed out by Sharpe (1994), strategies with a positive Sharpe Ratio should be held long, and those with negative Sharpe Ratio should be held short. In a situation of negative excess returns, the higher the volatility the higher (less negative) is the Sharpe Ratio. The problem in comparing funds with negative Sharpe Ratio—a situation that occurs often in bear markets—has been discussed in the literature (see for example Scholz and Wilkens, 2005). Despite this controversy, it does not mean that Sharpe ratio and derived measures are not suitable for measuring the abilities of active fund managers. We need only to be careful with the results. In fact, we have to keep in mind that higher Sharpe ratios are better than lower even in a bearish market, since we can optimally combine risk-free assets and the market portfolio.

Although both the Sharpe Ratio and ASSR suffer similar interpretation problems when excess returns are negative, there is an additional problem with ASSR. Note that equation (7) might generate imaginary numbers if the expression below the square root is negative. In fact, this does happen in our sample. Imaginary numbers may occur if the Sharpe Ratio is negative and the skewness positive, or vice-versa. Therefore, problems with ASSR are not restricted to bear markets. They might occur in an imaginary ASSR even with, for instance, a high positive Sharpe Ratio and a slightly negative skewness.

To solve the problem of negative numbers below the square root in equation (7) we suggest using a performance measure that emerges from equation(4), which is the allocation of the risky asset. In such an event, we would have no problems with imaginary numbers. Since all funds will have the same parameter $\gamma$, we suggest the following index for ranking managers (Adjusted for Skewness Performance Index – ASPI) that emerges from equation(4):

$$ASPI = \frac{SR}{\sigma\sqrt{\Delta t}}\left(1 + \frac{bS}{2}SR\right) \qquad (7)$$

We may thus rank fund managers in a manner similar to that of the ASSR, but without the problem of square roots of negative numbers. Negative skewness is penalized when excess returns are positive as in the ASSR. But how should we interpret negative ASPI? As the index is derived from the allocation of the risky asset, negative numbers mean we should short-sell the fund, in a situation similar to that of the Sharpe Ratio. As there are constraints on short-selling actively managed funds, we should refrain from investing in such funds, and put the money in the risky-free asset. If we must choose between two funds with negative ASPI, as happens with rankings, we would encounter the same problem of interpretation that occurs with the Sharpe Ratio. But, even in this case the rule is to choose the higher ASPI.

The *b* parameter works exactly as in the ASSR. The higher the value of *b*, the higher is the aversion to negative skewness. When *b* equals zero, ASPI is indifferent to skewness and is compatible with the use of a quadratic utility function with no penalty for skewness. If we use a CARA (constant absolute risk aversion) utility function, the value of *b* should be one (Koekebakker and Zakamouline, 2009). For a logarithmic utility, the value of *b* should be two.

If we use a CRRA (constant relative risk aversion) utility function, the value of *b* will depend on the relative risk aversion (RRA) parameter. For the Brazilian economy, Nakane and Soriano (2003) estimate values for the relative risk aversion from -0.1 to 4.3. For an RRA equal to one, we would have again *b* equal to two. If we consider an RRA value of three, *b* will be 4/3.

**4. Data**

Data was obtained from Bloomberg and consisted of daily Net Asset Values (NAV) of Multimarkets and Fixed-Income Brazilian Mutual funds from years 2003 to 2007, a total of 1255 working days. Classification in Multimarkets and Fixed-Income funds follows Andima (the National Association of Financial Market Institutions) standards, and fund of funds were excluded to avoid spurious clustering. The Multimarkets is a type of fund in Brazil that has characteristics similar to the international Hedge Funds. The risk-free asset used was the CDI (one-day interbank deposit).

The sample included only surviving funds, i.e., funds with data for the full period. We excluded from the sample funds those that did not have information about the NAV for more than five business days. For the missing days, the NAV was interpolated. We then had 375 funds: 186 Multimarkets and 189 Fixed-Income.

Given these exclusions, our sample of funds had a survivorship bias, i.e., funds that disappeared before the ends of the sample period were not included. But as our concern being to evaluate the ranking changes as a function of the performance measure, survivorship bias did not pose a problem. In fact, the only way to compare the performance of funds through a specific time interval was to use only funds that were alive through the entire period. However, one has to exercise care when analyzing the average performance of funds against a benchmark. As funds tend to be closed after negative results, the surviving funds will manifest an upward bias in their performance when compared with the whole sample.

## 5. Results

We calculated ASPI for each fund of the two samples of Brazilian funds with four values for the skewness preference parameter, *b*. We also calculated the Sharpe ratio for each fund of the two samples. Then we estimated the correlation for the results based on the Kendall's Tau Rank correlation. Our results are shown in Table 1. We see that correlation with the Sharpe Ratio is very low, except for the case where $b = 0$ (which means indifference to skewness). We can see also that as *b* increases, the correlation between ASPI and Sharpe ratio decreases.

On comparing our results with those of Eling and Schuhmacher (2007), we see a clear lower correlation in our study. Thus, the choice of Performance Measure actually does matter when we account for skewness.

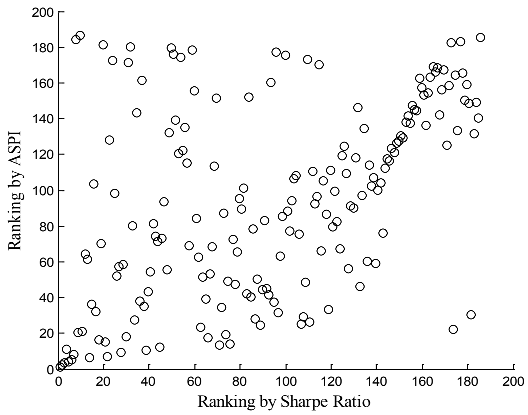
**Table 1**
**Kendall's Tau Rank Correlation Matrix**

| | | | *Multimarkets* | | |
|---|---|---|---|---|---|
| | ASPI with $b = 0$ | ASPI with $b = 1$ | ASPI with $b = 4/3$ | ASPI with $b = 2$ | Sharpe Ratio |
| ASPI with $b = 0$ | 1.00 | 0.66 | 0.58 | 0.44 | 0.64 |
| ASPI with $b = 1$ | 0.66 | 1.00 | 0.92 | 0.79 | 0.51 |
| ASPI with $b = 4/3$ | 0.58 | 0.92 | 1.00 | 0.87 | 0.45 |
| ASPI with $b = 2$ | 0.44 | 0.79 | 0.87 | 1.00 | 0.37 |
| Sharpe Ratio | 0.64 | 0.51 | 0.45 | 0.37 | 1.00 |
| | | | *Fixed Income* | | |
| | ASPI with $b = 0$ | ASPI with $b = 1$ | ASPI with $b = 4/3$ | ASPI with $b = 2$ | Sharpe Ratio |
| ASPI with $b = 0$ | 1.00 | 0.23 | 0.17 | 0.08 | 0.83 |
| ASPI with $b = 1$ | 0.23 | 1.00 | 0.93 | 0.84 | 0.22 |
| ASPI with $b = 4/3$ | 0.17 | 0.93 | 1.00 | 0.91 | 0.16 |
| ASPI with $b = 2$ | 0.08 | 0.84 | 0.91 | 1.00 | 0.08 |
| Sharpe Ratio | 0.83 | 0.22 | 0.16 | 0.08 | 1.00 |

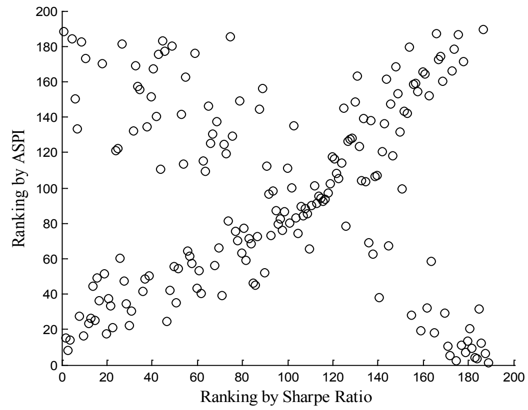
The following graphs show ranking comparisons. The best funds are those at the beginning of the axes. Sharpe Ratio rankings are shown on the X-axis and ASPI rankings on the Y-axis. We see that it is possible to calibrate investors' skewness preferences with the $b$ parameter. Higher negative skewness aversion may be modeled with a higher value of $b$ .

The graphs show a widely scattered pattern instead of a concentration along the 45$^{o}$ line which is expected if rankings were similar. Funds that are below the 45$^{o}$ line have been upgraded by the new measure when compared with the Sharpe Ratio.

**Ranking Sharpe Ratio X ASPI with *b*=2**

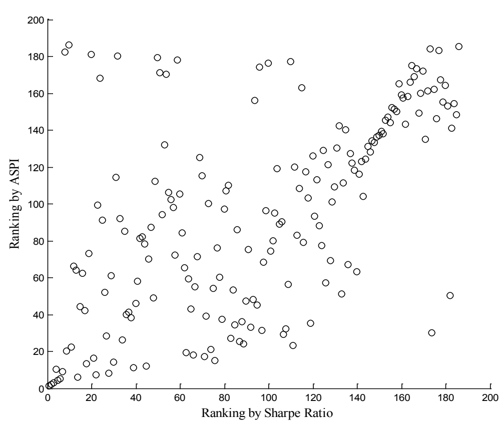



Multimarkets

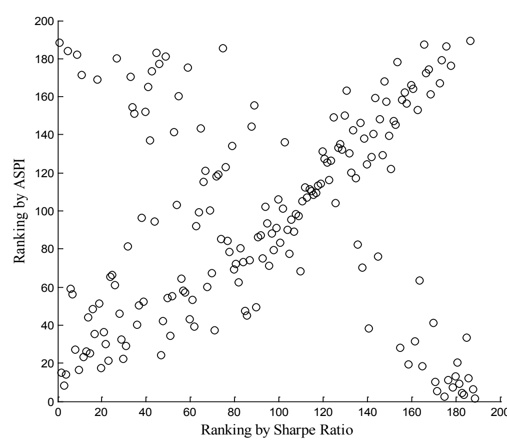



Fixed Income

**Ranking Sharpe Ratio X ASPI with b=4/3**

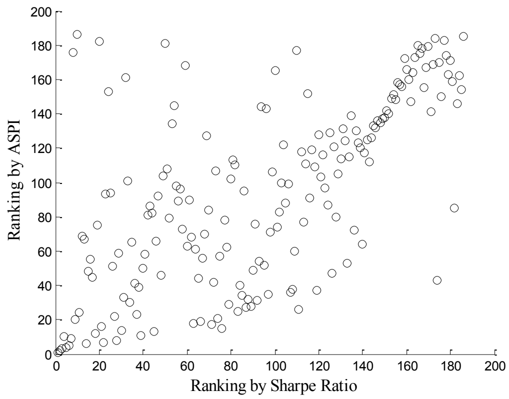



Multimarkets

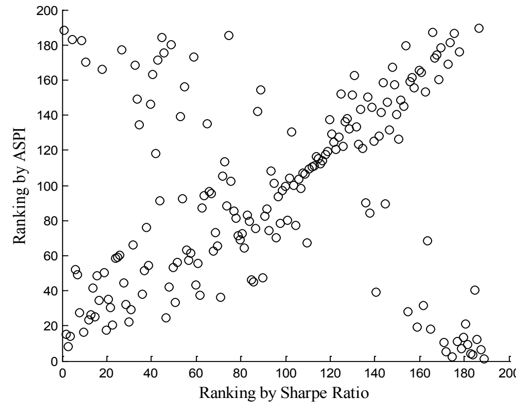



Fixed Income

**Ranking Sharpe Ratio X ASPI with b=1**

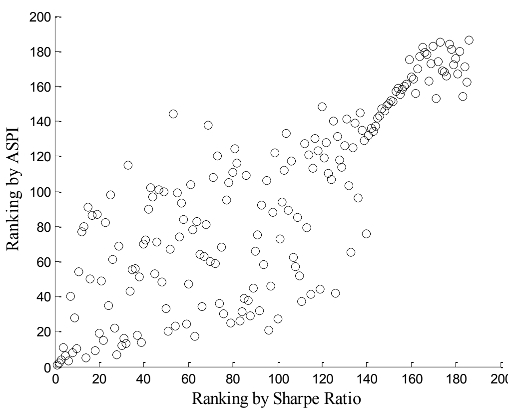


Multimarkets

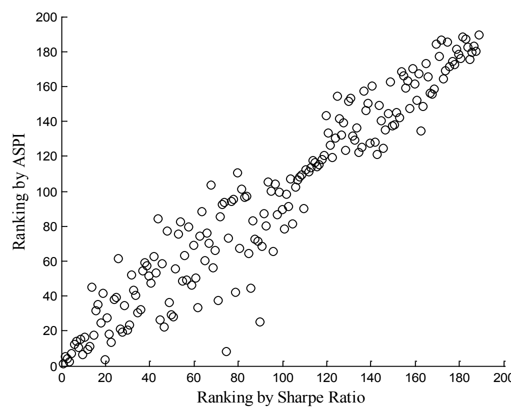


Fixed Income

**Ranking Sharpe Ratio X ASPI with b=0**



Multimarkets



Fixed Income

Conversely, funds that are above the 45º line have been downgraded by the new measure. One important aspect that needs to be analyzed is the statistical characteristics of these funds.

In order to identify and analyze these funds, we calculated for each fund the difference in rankings between the ASPI and the Sharpe Ratio. We then calculated the standard deviation of these differences in ranking and selected the funds that are over two standard deviations above or under the mean, i.e., outlier funds that have a Sharpe Ratio ranking much better or much worse than that of ASPI. For these funds, we calculated values for the Mean, Standard Deviation, Skewness and Kurtosis of excess returns. Table 2 shows the results. As expected, we see that funds that have been downgraded by ASPI have skewness far more negative than the overall mean skewness of the sample.

**Table 2**
**Characteristics of Funds well above or under the 45° Line**

| | # of Funds | Mean | Std | Skewness | Kurtosis |
|---|---|---|---|---|---|
| **ASPI with b=1** | | | | | |
| **Multimarkets** | | | | | |
| | # of Funds | Mean | Std | Skewness | Kurtosis |
| SR gives Better Rank than ASPI | 7 | 0.0078 | 0.0056 | -2.2464 | 69.4714 |
| ASPI gives Better Rank than SR | 2 | -0.0156 | 0.0241 | 9.0681 | 319.4699 |
| Overall Sample | 186 | 0.0138 | 0.0336 | -0.3358 | 61.3812 |
| **Fixed-Income** | | | | | |
| | # of Funds | Mean | Std | Skewness | Kurtosis |
| SR gives Better Rank than ASPI | 6 | 0.0048 | 0.0015 | -2.6002 | 70.6551 |
| ASPI gives Better Rank than SR | 15 | -0.0321 | 0.0012 | 6.3231 | 204.5035 |
| Overall Sample | 189 | -0.0036 | 0.0077 | -0.0618 | 199.3851 |
| **ASPI with b=4/3** | | | | | |
| **Multimarkets** | | | | | |
| | # of Funds | Mean | Std | Skewness | Kurtosis |
| SR gives Better Rank than ASPI | 9 | 0.0072 | 0.0054 | -2.1715 | 61.7502 |
| ASPI gives Better Rank than SR | 2 | -0.0156 | 0.0241 | 9.0681 | 319.4699 |
| Overall Sample | 186 | 0.0138 | 0.0336 | -0.3358 | 61.3812 |
| **Fixed-Income** | | | | | |
| | # of Funds | Mean | Std | Skewness | Kurtosis |
| SR gives Better Rank than ASPI | 6 | 0.0048 | 0.0015 | -2.6002 | 70.6551 |
| ASPI gives Better Rank than SR | 15 | -0.0321 | 0.0012 | 6.3231 | 204.5035 |
| Overall Sample | 189 | -0.0036 | 0.0077 | -0.0618 | 199.3851 |
| **ASPI with b=2** | | | | | |
| **Multimarkets** | | | | | |
| | # of Funds | Mean | Std | Skewness | Kurtosis |
| SR gives Better Rank than ASPI | 11 | 0.0069 | 0.0052 | -1.9667 | 59.6465 |
| SR gives Better Rank than MPPM | 2 | -0.0156 | 0.0241 | 9.0681 | 319.4699 |
| Overall Sample | 186 | 0.0138 | 0.0336 | -0.3358 | 61.3812 |
| **Fixed-Income** | | | | | |
| | # of Funds | Mean | Std | Skewness | Kurtosis |
| SR gives Better Rank than ASPI | 5 | 0.0048 | 0.0013 | -2.6972 | 66.4838 |
| SR gives Better Rank than MPPM | 14 | -0.0343 | 0.0012 | 6.4637 | 214.7075 |
| Overall Sample | 189 | -0.0036 | 0.0077 | -0.0618 | 199.3851 |

Another way to assess the reasons for these differences is to regress the skewness on the Sharpe Ratio and ASPI rankings. The results on Table 3 confirm that the funds whose performance with ASPI is better have higher skewness.

**Table 3**
**Regression of Skewness on Sharpe Ratio and ASPI Rankings**

| Equation: Skewness = $\alpha + \beta_1$ SR_Ranking + $\beta_2$ ASPI_Ranking | | | | | |
|---|---|---|---|---|---|
| | $\alpha$ | SR | ASPI | $R^2$ | F Test |
| Multimarkets | 2.3120*** | 0.0068 | -0.0351*** | 0.248 | 30.17*** |
| Fixed-Income | 11.8247*** | -0.3620 | -0.1099*** | 0.371 | 54.83*** |

*** Coefficient significant at 1%

## 6. Conclusion

This paper proposes and uses a performance evaluation measure that takes into account skewness. This approach may be useful for investors who are averse to negative skewness.

Our measure is based on the ASSR (Adjusted for Skewness Sharpe Ratio) proposed by Koekebakker and Zakamouline (2009). We show that despite the intuitiveness of their measure, ASSR is not suitable for ranking funds due to the possible presence in the measure of the square roots of negative numbers. In fact, this occurred several times in our sample of Brazilian funds. Therefore, we argue that an appropriate measure for ranking funds would be the amount of investment their model allocates to the risky asset, based on investors' utility function. We call this new measure Adjusted for Skewness Performance Index (ASPI) and used it to rank Brazilian Fixed Income and Multimarkets funds.

The results of the ASPI measure for our sample of Brazilian funds show very low correlations with Sharpe ratio rankings. It is therefore clear that when we take into account skewness, rankings may differ appreciably from traditional approaches. Obviously, the choice of this type of measure depends on investors' preferences for skewness. Those who are apprehensive about large losses may adjust their preferences with a parameter that tunes skewness preferences.

One common limitation in empirical performance evaluation researches is that the length of the sample period influences the results, and this is the case in this paper. In fact, we use a limited time period for our sample. Thus, for future research, we suggest the use of different time windows and investment horizons in the performance evaluation, in order to test robustness of results.

Another limitation of our paper is that, although our approach allows a wide range of utility functions[1], when we set the value of parameter *b*, we are tied to a specific utility function. While this feature of our approach may be useful for performance evaluation of specific investors, it creates problems if one wants a general ranking of funds, since different investors may have different preferences. However, this preference-dependence is a characteristic of many performance indexes. The Sharpe Ratio, for instance, assumes a quadratic preference.

There is no consensus in performance evaluation approaches for ranking funds. Measures will provide different rankings that should be interpreted by investors with care. It does not appear prudent to rely on a single measure to choose the fund to invest your money. So, it is necessary to understand performance measurement theory, strengths and weaknesses of indexes, and investors' preferences regarding risks and returns, for appropriate allocation of wealth.

[1] As seen before, if we use a CARA (constant absolute risk aversion) utility function, the value of **b** should be one. For a logarithmic utility, the value of **b** should be two. If we use a CRRA (constant relative risk aversion) utility function, the value of **b** will depend on the relative risk aversion parameter

**References**

Almeida, Mário Augusto, 2004. Análise das Preferências dos Investidores: Uma Análise de Dados em Painéis. *Proceedings of the IV Meeting of the Brazilian Finance Society*.

Alves Junior, Antonio José, 2003. Fundos Mútuos de Investimento no Brasil – A Expansão da Indústria nos Anos 1990 e Perspectivas para o Futuro. *Working Paper Cepal*.

Berk, J.B. And R.C. Green, 2004. Mutual Fund Flows and Performance in Rational Markets. *Journal of Political Economy* 112, 1269-1295.

Cerny, Ales, 2003. Generalised Sharpe Ratios and Asset Pricing in Incomplete Markets. *European Finance Review* 7, 191-233.

Dowd, K., 2000. Adjusting for Risk: An Improved Sharpe Ratio. *International Review of Economics and Finance* 9, 209–222.

Dynking, Lev, Gould, Anthony, Hyman, Jay, Konstantinovsky, Vadim and Phelps, Bruce, 2007. *Quantitative Management of Bonds Portfolios*. Princeton University Press.

Eid Jr, William and Rochman, Ricardo, 2005. Medidas de Desempenho de Fundos Considerando Risco de Estimação. *Proceedings of the V Meeting of the Brazilian Finance Society*.

Eling, M. and Schuhmacher, F., 2007. Does the choice of performance measure influence the evaluation of hedge funds?. *Journal of Banking & Finance* 31, Issue 9, 2632-2647.

Elton, Edwin, Gruber, Martin and Blake, Christopher, 2003. Incentive Fees and Mutual Funds. *The Journal of Finance* 58, n. 2, 779-804.

Goetzmann, William, Ingersoll, Jonathan, Spiegel, Matthew and Welch, Ivo, 2007. Portfolio Performance Manipulation and Manipulation-proof Performance Measures. *The Review of Financial Studies* 20, N. 5, 1503-1546.

Grava, William and Siqueira, José, 2003. O Preço da Volatilidade como Índice de Avaliação de Performance de Fundos de Investimento em Ações: Uma Recompensa ao Conservadorismo. *EAD/FEA/USP Working Papers 03/008*.

Hodges, S. ,1998. A Generalization of the Sharpe Ratio and its Applications to Valuation Bounds and Risk Measures. *Working Paper - Financial Options Research Centre*, University of Warwick.

Henriksson, R. D., and R. C. Merton, 1981. On Market Timing and Investment Performance. II. Statistical Procedures for Evaluating Forecasting Skills, *Journal of Business* 54, 513–533.

Jensen, M., 1969. Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios. *Journal of Business* 42, 167–247.

Keating C. and Shadwick, W., 2002. A Universal Performance Measure. *Journal of Performance Measurement* 6, 59-84.

Koekebakker, Steen and Zakamouline, Valeri, 2009. Portfolio performance evaluation with generalized Sharpe ratios: Beyond the mean and variance. *Journal of Banking and Finance* 33, 1242-1254.

Lynch, Anthony and Musto, David, 2003. How Investors Interpret Past Fund Returns. *The Journal of Finance*, v. LVIII, n. 5, 2033-2058.

Martins, Fábio ,2006. Análise de Desempenho dos Fundos Mútuos de Gestão Ativa: O Índice de Sharpe Probabilístico. *Proceedings of the VI Meeting of the Brazilian Finance Society*.

Malkiel, B. and Saha, A., 2005. Hedge Funds: Risk and Return. *Financial Analysts Journal* 61, Number 6, 80-88.

Nakane, Márcio and Soriano, L., 2003. Real Balances in the Utility Function: Evidence for Brazil. *Working Paper* No. 68, *Banco Central do Brasil*.

Oliveira, Rodrigo, 2005. Desempenho, Persistência dos Retornos e Captação na Indústria de Fundos Multimercados no Brasil. *Master Thesis – UFRJ.*

Scholz, Hendrik and Wilkens, Marco, 2005. Interpreting Sharpe Ratios - The Market Climate Bias, *Working Paper available at SSRN*: http://ssrn.com/abstract=524842

Sharpe, W. F., 1966. Mutual Fund Performance. *Journal of Business* 39, 119–138.

Sharpe, W. F., 1994. The Sharpe Ratio. *The Journal of Portfolio Management*, 21 (1), 49-58.

Silva Junior, A. F., 2004. Performance Attribution for Fixed Income Portfolios in Central Bank of Brazil International Reserves Management. In *Risk Management for Central Bank Foreign Reserves*. Edited by European Central Bank.

Sortino, F. and Van Der Meer, R., 1991. Downside Risk. *Journal of Portfolio Management* 17, Summer, 27–31.

Sortino, F., Van Der Meer, R. and Plantinga, A., 1999. The Dutch Triangle. *Journal of Portfolio Management* 26 (1), pp. 50-59.

Sortino, F. A. and Price, L. N., 1994. Performance Measurement in a Downside Risk Framework. *Journal of Investing* 3, 59 – 65.

Stutzer, M., 2000 A Portfolio Performance Index, *Financial Analysts Journal* 56 (3), 52 – 61.

Treynor, J., 1965. How to Rate Management of Investment Funds. *Harvard Business Review* 43, 63–75.

Treynor, J., and Mazuy, K., 1966. Can Mutual Funds Outguess the Market?. *Harvard Business Review* 44, 131–136.

Vinod, H.D. and Morey, M.R., 2004. A "Double" Sharpe Ratio. *SSRN Working Paper 1999b*; http://ssrn.com/abstract=168748.