Modeling Bank Failure Risk

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In this study we develop a notional model that assesses the creditworthiness of a conventional bank. The theory first segregates several independent risk components among typical banking institutions and classifies these modules as momentarily "hedged" or "exposed." The proposed model then evaluates the instantaneous failure probabilities attached to each risk component by analyzing its observable stochastic behavior. These inferences are further used to compose the complete risk profile of a bank and to derive numerous failure probabilities, depending on the relevant economic setting. While prior studies have associated the creditworthiness of financial institutions to specific economic variables, ordinarily one at a time, our unique contribution resides within the construction of a more comprehensive scheme, which aggregates several risk modules into a single measure of bank failure. We suggest four alternative estimation techniques for the model parameters, and further demonstrate the model's predictive strength with genuine examples.

JEL Classifications: G01, G21, G28, G33

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1. Introduction

There are few occasions in the history where bank failures are clustered in colossal numbers. The stock market crash in 1929 is often cited as being the beginning of the great depression. Over 700 banks failed in that year, and more than 9,000 banks failed throughout the following decade. The savings and loan crisis of the late 1980s and early 1990s brought down about 750 commercial banks and other loan associations. The late 2000s financial crisis has led thus far to almost 300 failed U.S. banks, yet many expect this figure to rise in the coming years.[1] Banks play a prominent role in a nation's economic life, and the event of a bank failure touches nearly every aspect of the financial markets. The ability to accurately model bank credit risk has become among the top concerns of regulators, governmental agencies, insurance companies, financial institutions, corporate lenders, small businesses, and private investors.

In this article we propose a notional model that assists in constantly monitoring operative financial enterprises and helps to predict bank failures. We propose a theory that approximates the chances for a bank failure by uncovering the stochastic processes of different risky components within typical bank activities. Explicitly, we assess the likelihoods for conjectural jumps in commercial real estate and construction lending, leasing finance, agricultural lending, accounts receivable and inventory financing, retail credit, foreign assets, other loans purchased at discount, and investment securities and derivatives. We then combine these risky modules and estimate general bank failure likelihoods under various economic scenarios: when the lending institution is fully exposed or partially hedged to these eight competing risks. We further suggest four alternative estimation guidelines for the model parameters and demonstrate the scheme with empirical examples. Our theory is inclusive and can fit different banking institutions in various markets. Nevertheless, it is also pertinent and useful. It can be modified according to explicit circumstances and match diverse economic settings.

[1] A Wall Street Journal article from Sep. 27th, 2010, named "Banks Keep Failing, No End in Sight," quotes a recent report by Standard & Poor's as holding this dark vision.

Our strategy for examining bank failure risk fundamentally diverges from prior theoretical models and empirical investigations on this matter. Previous studies have associated the creditworthiness of financial institutions to specific economic variables, typically one at a time. These financial and legal aspects include the relative size of marketable securities within a bank's portfolio, the significant regulatory enforcement measures taken, the variability of the economic cycles, the relevant individual and consumer loans, the respective credit spread curves, the pertinent agency conflicts, the ad hoc household credit developments, and the general exposure to interest rate risk. We, on the other hand, construct a comprehensive analytical scheme through a close examination of the immediate probabilities of the risk-adjusted stochastic assets that may cause a fundamental failure within a bank portfolio. We then aggregate the different risk components into a single measure of default likelihood under various economic settings. We therefore contribute to the literature by offering a rigorous yet highly applicable credit failure model for banks and financial institutions alike. We further authenticate our scheme with valid records of both failed and non-failed banks, and confirm its superior predictive strength over relatively naïve approaches that merely examine RBC ratios.

Banks and other depository institutions that are unable to service their outstanding debt or incapable of sustaining the Risk Based Capital (RBC) adequacy minimum ratios are typically classified as "failed banks." However, in most developed countries the bankruptcy codes do not allow insolvent financial institutions to file for bankruptcy. Instead, domestic chartering authorities order failed banks into receivership, while the Federal Deposit Insurance Corporation (FDIC) is the appointed receiver in the U.S.[2]

The FDIC routinely monitors the capital adequacy of banks by examining a large set of risk factors including the past and the current financial condition, managerial resources, future earnings prospects, the magnitude of off-balance sheet and funding risks that often include derivatives and foreign exchange contracts, concentration related risks, unsafe or unsound practices, letters of credit, the level of interest rate risk exposure, other nontraditional bank activities, Tier 1 and Tier 2 capitals, unfunded loan commitments, and classification of assets as substandard, doubtful, or otherwise criticized.[3]

The FDIC aggregates these risk components and further computes for each inspected bank three common capital ratios as follows: (1) Tier 1 capital adequacy ratio as Tier 1 capital divided by risk-adjusted assets, (2) total capital ratio as Tier 1 and Tier 2 combined capitals divided by risk-adjusted assets, and (3) leverage ratio as Tier 1 capital divided by the average total consolidated assets.[4] Banks constantly report these capital ratios on the Call Report or the Thrift Financial Report.

Following the principal architecture of the three Basel Capital Accords, the Board of Governors of the Federal Reserve System (FRB) dictates the risk-based capital standards as the lower thresholds for the RBC adequacy ratios. The minimum requirements can vary from time to time, but the Tier 1 capital ratio normally ranges from four to six percent, the total capital ratio usually varies from eight to ten percent, and the leverage ratio commonly stretches from four to five percent.

Whenever a bank breaches these minimum required RBC adequacy ratios, the FDIC assumes control over the bank's receivership assets and liabilities and acts as the insurer of all insured deposits for banks that are chartered by the federal government, as well as for most state chartered

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[2] Typically, U.S. banks are not subject to actual closure by their chartering authority until their leverage ratios, which include tangible common and preferred equity-to-book assets, under prompt corrective action rules, fall below two percent. Even then, banks could request up to 270 days of extensions of the closing to arrange friendly mergers. Nevertheless, the model hereafter considers the FDIC classification of "failed bank" and not necessarily the threat of ultimate bank closure. The current scheme further disregards conceivable government bailouts. These are uncontrollable variables to any theory of this kind.

[3] Tier 1 bank capital consists largely of common shareholders' equity and is considered as a permanent capital. In contrast, Tier 2 bank capital is composed of undisclosed reserves, revaluation reserves, general provisions, and subordinated-term debt. Tier 2 bank capital is therefore considered as a temporary or a supplementary capital.

[4] A Wall Street Journal article from Sep. 1st, 2010, named "FDIC Finds 829 U.S. Banks at Risk," reveals that more than tenth of U.S. banks are still at risk of failure, although many "boosted their results by setting aside less to cover future loan losses than they have in recent quarters."

banks. Yet, throughout the past century Congress and federal courts have granted the FDIC exceptional legal rights to (1) disregard a failed bank's prior agreements such as loan modifications, (2) repudiate existing contracts, including leases, letter of credit draws, and service contracts, (3) temporarily suspend pending litigation or administrative proceedings for up to 90 days, (4) extend statues of limitations on different claims, and (5) reinforce some statues of limitations that have already expired. The FDIC is not the only one to enjoy these special legal rights. Most financial institutions including joint ventures, trustees, and private investors that purchase loans from failed banks would benefit from these distinctive privileges.

Consequently, failed banks often cause significant personal and corporate losses and occasionally trigger broader macroeconomic setbacks. Acquiring banks are required by law to have sufficient capital to absorb the failed banks' assets and liabilities without becoming undercapitalized. As a result, acquiring banks are sometimes incompetent in originating new loans. This "recycling of assets" process conveys significant implications for the broad economy, mainly in the form of a substantial decline in bank lending activity, which particularly depresses private investors and small businesses. Therefore, it is vital to accurately assess bank credit risk in advance from both personal and social viewpoints. In this study we attempt to enhance the predictive power towards bank failures among policymakers, bank examiners, and other market participants.

This research proceeds as follows. In Section 2 we review some of the more prominent advances in bank credit risk modeling. In Section 3 we present a theoretical scheme that evaluates the creditworthiness of a conventional bank. In Section 4 we provide some estimation guidelines. In Section 5 we empirically illustrate the model predictions, and in Section 6 we conclude. This study uses many notations, thus we review them in Appendix 1. In Appendix 2 we grant further intuition for the theory.

2. Literature Review

Our attitude to consider a bank's total probability of insolvency as an integrated scheme of all of its risk-adjusted sources agrees with Mingo's (2000) perception that banks can mask capital weakness although achieving the regulatory RBC adequacy ratios. On the other hand, we focus on the key risk-adjusted bank activities rather than examining the abundant individual and consumer bank loans and their corresponding internal ratings, since the latter approach is overly complex and often ineffective, as discussed by Domowitz and Sartain (1999), Treacy and Carey (2000), and Gross and Souleles (2002). Kuritzkes and Schuermann (2008) provide an intriguing philosophical view on this matter.

Numerous scholars have attempted alternative approaches to assess commercial bank credit risk exposure and offer various explanatory variables for bank creditworthiness. Dothan and Williams (1980) provide theoretical arguments that a banks' probability of failure is mainly composed of its relative size of portfolio of marketable securities. Peek and Rosengren (1995) investigate how regulatory enforcement actions affect banks' lending behavior and as a result banks' survivability. Kaminsky and Reinhart (1999) associate bank failures to economic cycles. The authors show that banks tend to fail as the economy enters a recession, habitually following an extended period of elevated bank lending activity.

Hellmann, Murdock, and Stiglitz (2000) explain that banks have higher incentive to take further risk when their franchise value is harmed. Thus, their creditworthiness exhibits different patterns in expansionary and contractionary economic phases. Barnhill and Maxwell (2002) simulate both credit risk and market risk for a complete bank portfolio and its stability to macroeconomic variations. Jacobson and Roszbach (2003) explore how marginal changes in a default-risk-based acceptance rule would affect a bank's Value at Risk (VaR) exposure and its expected credit losses.

Krishnan, Ritchken, and Thomson (2004) examine whether banks' credit-spread curves can forecast banks' risk. The authors find that the slopes of these curves are significant predictors of future credit spreads, and further realize that this relationship is more pronounced among relatively small and highly leveraged banks, and among banks with high levels of net-charge-off. Liao, Chen, and Lu (2009) use U.S. banking data from 2001 to 2005 to empirically demonstrate the impact of agency problems and information asymmetry between shareholders and debt-holders on the evaluation of bank credit risk.

Marcucci and Quagliariello (2009) use threshold regressions to validate the asymmetric effects of business cycles on bank credit risk. Büyükkarabacak and Valev (2010) show that household credit expansions have been statistically and economically significant predictors of banking crises across both developed and emerging markets. Drehmann, Sorensen, and Stringa (2010) theoretically derive an integrated scheme of bank credit and interest rate risks by considering the common merits of assets, liabilities, and specific off-balance sheet items.

In a series of articles, Moody's (1999, 2003, and 2007) presents a recommended list of quantitative and qualitative factors that affect bank creditworthiness. Overall, these risk components are clustered into seven pillars of bank analysis as follows: (1) operating and regulatory environments, (2) ownership and governance, (3) franchise value, (4) earning power and financial fundamentals, (5) risk profile, (6) economic capital analysis, and (7) management priorities and strategies. Moody's further provides suggestions on how to examine the specific risk components, yet this framework remains a knowledge-based expert system, which leaves most of the decisions in the hands of credit analysts.

Our approach radically differs from prior work on bank credit risk. We construct a complete analytical model to assess the viability of financial institutions through a close examination of the instantaneous probabilities of the risk-adjusted stochastic assets that can cause a fundamental failure within a bank portfolio. We then aggregate the different risk modules into a single measure of default likelihood under various economic settings. We contribute to the literature by offering a comprehensive yet highly applicable credit risk model for banks and financial enterprises alike. We further authenticate our scheme with valid records of both failed and non-failed banks, and confirm its supremacy over naïve approaches that only examine RBC ratios.

3. The Model

The U.S. Office of the Comptroller of the Currency (OCC) has identified in the Comptroller's handbook (2001) eight central bank activities that can serve as independent quantitative risk-adjusted sources.[5] These are (1) commercial real estate and construction lending, (2) leasing finance, (3) agricultural lending, (4) accounts receivable and inventory financing, (5) retail credit, (6) foreign assets, (7) other loans purchased at discount, and (8) investment securities and derivatives. These common risk-adjusted bank assets are subject to stochastic processes over time, and since they progress from disjoint origins, they are further assumed to be independent of one another.[6]

Many commercial banks operate with RBC ratios that are only marginally higher than the regulatory lower thresholds.[7] Therefore, we can further assume that each source of risk by itself, unless perfectly hedged, could cause a bank's RBC ratios to fall below the minimums required by law, thus independently force a bank failure.[8] We denote the sovereign magnitudes of these risk-

[5] For purpose of model tractability we do not consider qualitative risk factors such as managerial resources or potential regulatory actions as a result of unsafe or unsound practices. However, we can properly adjust the model whenever these qualitative attributes are transformed into quantitative measures.

[6] There could be various inter-dependencies within these risk-adjusted bank assets. For example, loan write-downs can result in the recovery of collateral. Hence, decreases in loan values may trigger increases in the asset category containing repossessed assets. Furthermore, banks that decide to fund real estate development projects after completion reclassify loans from development to permanent structure loans. Nevertheless, these inter-dependencies do not affect the sovereign structure across the eight different risk-adjusted bank activities.

[7] Dionne and Harchaoui (2003) report that from 1988 to 1998 Canadian banks had median Tier 1 capital ratios ranging from 5.3% to 8.9%, compared to the minimums required of four to six percent. During this period Canadian banks had median total capital ratios varying from 8.3% to 11.3%, compared to the lower regulatory thresholds of eight to ten percent. In the later empirical section we further confirm this presumption with U.S. commercial banks from 1990 to 1993.

[8] Throughout this study we use the term "hedge" to represent zero exposure to a specific risk source. It does not necessarily mean the traditional use of financial derivatives. It can also be the end result of a continuous effort to minimize a particular bank activity.

adjusted assets as $M_i,\ i \in \{1,..,8\}$.

There are numerous ways to model the stochastic behavior of each risk-adjusted source.[9] Among them we refer to a simple Geometric Brownian Motion (GBM), a GBM with Poisson jumps as in Merton (1976), an independent square-root process with stochastic volatility as in Heston (1993), a stochastic volatility with jumps as in Bates (1996), or a longer memory stochastic process with jumps perturbed by fractional noise as in Sattayatham, Intarasit, and Chaiyasena (2007).

Since we are interested in finding the instantaneous bank failure probabilities associated with sudden shifts in the magnitude of the risk-adjusted assets, we favor Merton's (1976) approach for occasional discontinuous breaks within the risk-adjusted sources.[10] In this case, the Stochastic Differential Equations (SDE) for the sovereign magnitudes of the eight risky assets are:

$$dM_{i,t} = \mu_i dt + \sigma_i dW_{i,t} + \omega dJ_{i,t} \qquad i \in \{1,..,8\}, \tag{1}$$

where μ denotes the process's drift, σ is the process's diffusion, W represents a standard Wiener process, dJ is a Poisson counter with intensity λ, and ω depicts a draw from a Normal distribution, hence $\omega \sim N(\mu_J, \sigma_J^2)$. The Poisson jump-diffusion processes are leptokurtic and can be skewed. We further define the periodic logarithmic corrections for the eight independent magnitudes as:

$$\chi_{t+1} \equiv \ln\left(\frac{M_{i,t+1}}{M_{i,t}}\right) = \begin{cases} \upsilon & \text{if N} = 0 \\ \upsilon + \sum_{j=1}^{N} \omega_j & \text{if N} \geq 1 \end{cases}, \tag{2}$$

where υ is a draw from the respective GBM process, and ω_j are draws from the jump process. The likelihoods for the corresponding instantaneous stochastic jumps are set with respect to the parameters of the Poisson processes as:

$$P(N = n) = \frac{e^{-\lambda}\lambda^n}{n!}. \tag{3}$$

Therefore, we can define the probability δ for a single instantaneous stochastic jump that could independently cause a bank failure at a specific time τ as $\delta(\tau, i) \equiv P(N = 1) = e^{-\lambda}\lambda$. Since the eight risk-adjusted assets are assumed to be independent of one another, we can aggregate them into a total probability for a bank failure Ψ at a specific time τ within the time interval $[t, t+1]$ as:[11]

$$\sum_{i=1}^{8} \delta(\tau, i) \equiv \Psi(\tau). \tag{4}$$

Similar to VaR and default risk universal analyses, for most practical purposes we snall examine the chances for a bank failure over time intervals of ten days forward. We define this time frame as a standard time-unit thus we focus hereafter on the time periods of $[t, \tau, t+1]$.[12] In this framework the following theory requires one further assumption. While the probability $\delta(\tau, i)$ for a single instantaneous stochastic jump in the magnitude of a risk-adjusted asset M_i is a function of the specific time τ, we assume that the ratio $\delta(\tau, i)/\Psi(\tau)$ is uniformly distributed within the interval

[9] Risk-adjusted bank components are typically reported to regulators once every quarter of a year, yet internal risk officers can sample these figures more frequently, thus obtain stochastic continuous patterns.

[10] Duffee (1999) utilizes a risk-free interest rate dynamic as an appropriate benchmark and chooses the independent square-root stochastic process to describe instantaneous corporate default risk.

[11] Implicitly, we assume that bank specific risk exposures are additive due to their sovereign nature.

[12] Jorion (1988) and Bates and Craine (1999) use parallel time intervals of ten days. Nevertheless, for the model to be applicable, practitioners may define any other time frame as a standard time-unit, as long as it is relatively short. The later assumption of constant relative risk proportions compels this provision.

$t,t+1$, thus it is independent of the specific time τ throughout this time frame. Instead, this measure of a relative risk proportion is a function of the complete time period $t,t+1$ and the magnitude of the relevant risk-adjusted source M_i.

The economic intuition for this assumption relies upon the reasonably short time frame $t,t+1$ and the fairly large number of independent risk-adjusted bank assets. We allow the instantaneous failure probability $\delta\ \tau,i$ to momentarily adjust, but command a constant relative proportion during the respective standard time frame. Thus, we define

$$\frac{\delta\ \tau,i}{\Psi\ \tau} \equiv \eta\ t,i\ . \qquad (5)$$

In Appendix 2 we demonstrate that a sufficient (but not a necessary) condition for this assumption of constant relative proportion holds whenever the underlying continuous distributions of the stochastic jumps in the risk-adjusted assets follow either the general Weibull distribution or the more specific Exponential distribution, as derived from the Poisson jumps within the stochastic processes of the risk-adjusted bank assets.

The analysis hereafter builds upon the competing risks theory, a singular mathematical branch that investigates multiple failure-origins. We are particularly interested to explore several characteristics of different positive probabilities for a bank failure as follows:

$\alpha\ t =$ the probability that an operating bank at time *t* will continue to be operative (will not fail) during the standard time interval $t,t+1$,

$\beta\ t =$ the probability that an operating bank at time *t* will fail during the time interval $t,t+1$ without identifying any specific stochastic jump causing this failure,

$\varepsilon\ t,i =$ the probability that an operating bank at time *t* will fail during the time interval $t,t+1$ due to a stochastic jump in the magnitude of a specific risk-adjusted asset M_i when all other risk-adjusted assets $M_j, j \in\ 1,..8\ , j \neq i$ are not hedged,

$\gamma\ t,i =$ the probability that an operating bank at time *t* will fail during the time interval $t,t+1$ due to a stochastic jump in the magnitude of a specific risk-adjusted asset M_i when all other risk-adjusted assets $M_j, j \in\ 1,..8\ , j \neq i$ are perfectly hedged,

$\rho\ t,i =$ the probability that an operating bank at time *t* will fail during the standard time interval $t,t+1$ when M_i is perfectly hedged,

$\pi\ t,i,j =$ the probability that an operating bank at time *t* will fail during the standard time interval $t,t+1$ due to a stochastic jump in the magnitude of a specific risk-adjusted asset M_i when a single risk-adjusted asset $M_{j\neq i}$ is perfectly hedged, and

$\pi\ t,i,j,k =$ the probability that an operating bank at time *t* will fail during the time interval $t,t+1$ due to a stochastic jump in the magnitude of a specific risk-adjusted asset M_i when two other risk-adjusted assets $M_{j\neq i}$ and $M_{k\neq i}$ are perfectly hedged.

Since the stochastic Poisson jump process governs the discrete instantaneous probabilities for a bank failure, the continuous Exponential likelihood for an operating bank at time *t* to remain fully operative throughout the time interval $[t, t+1]$ is:

$$\alpha(t) = \exp\left[-\int_{t}^{t+1} \Psi(\tau)\, d\tau\right], \tag{6}$$

and the complement probability for a bank failure is $\beta(t) = 1 - \alpha(t)$. In the special case when the total probability for a bank failure is constant throughout the time interval $[t, t+1]$, explicitly when $\Psi(\tau) = \Psi$, we obtain $\beta(t) = 1 - \exp(-\Delta\Psi)$, where Δ designates the length of the interval $[t, t+1]$. When $\Delta \to \infty$, $\beta(t) \to 1$, and when $\Delta \to 0$, $\beta(t) \to 0$. Similarly, when a bank is exposed merely to a single risk-adjusted asset M_i, explicitly when all other risky components are perfectly hedged, the respective likelihood for a bank failure becomes:

$$\gamma(t,i) = 1 - \exp\left[-\int_{t}^{t+1} \delta(\tau,i)\, d\tau\right] \quad \forall i \in \{1,..8\}. \tag{7}$$

Moreover, we can apply the multiplication and addition properties of integrals and postulate the probability of a bank failure from a stochastic jump in the magnitude of a specific risk-adjusted asset M_i when all other risky components within the bank portfolio are present as:

$$\varepsilon(t,i) = \int_{t}^{t+1} \exp\left[-\int_{t}^{\tau} \Psi(\tilde{\tau})\, d\tilde{\tau}\right] \delta(\tau,i)\, d\tau \quad \forall i \in \{1,..8\}. \tag{8}$$

The inner integral denotes the probability for a bank to remain operative from *t* to τ when the bank is exposed to all risky assets, whereas the outer integral designates the instantaneous likelihood for a bank failure from a specific risk-adjusted source M_i at a precise point in time τ within the time interval $[t, t+1]$. We reorganize equation (8) by utilizing the assumption for a stable proportional risk as expressed in (5) and obtain:

$$\begin{aligned} \varepsilon(t,i) &= \frac{\delta(\tau,i)}{\Psi(\tau)} \int_{t}^{t+1} \exp\left[-\int_{t}^{\tau} \Psi(\tilde{\tau})\, d\tilde{\tau}\right] \Psi(\tau)\, d\tau \\ &= \frac{\delta(\tau,i)}{\Psi(\tau)} \left\{1 - \exp\left[-\int_{t}^{t+1} \Psi(\tau)\, d\tau\right]\right\} = \frac{\delta(\tau,i)}{\Psi(\tau)} \left[1 - \alpha(t)\right] = \eta(t,i)\, \beta(t) \end{aligned} \tag{9}$$

We thus conclude that $\delta(\tau,i)/\Psi(\tau) = \varepsilon(t,i)/\beta(t)$ for all $i \in \{1,..8\}$ within the standard time interval $[t, \tau, t+1]$. Hence, the fixed proportional instantaneous risk instructs the ratio of the corresponding bank failure probabilities throughout the respective time frame. We can further deduce that $\sum_{i=1}^{8} \varepsilon(t,i) = \beta(t)$.

We use equation (6) and property (9) and reorganize equation (7) as follows:

$$\begin{aligned} \gamma(t,i) &= 1 - \exp\left[-\frac{\delta(\tau,i)}{\Psi(\tau)} \int_{t}^{t+1} \Psi(\tau)\, d\tau\right] = 1 - \exp\left[-\frac{\varepsilon(t,i)}{\beta(t)} \int_{t}^{t+1} \Psi(\tau)\, d\tau\right]. \\ &= 1 - \left[\alpha(t)\right]^{\varepsilon(t,i)/\beta(t)} = 1 - \left[\alpha(t)\right]^{\eta(t,i)} \quad \forall i \in \{1,..8\} \end{aligned} \tag{10}$$

By using the same approach we can develop the likelihood for an operating bank at time *t* to fail during the standard time interval $[t, t+1]$ when a single risk-adjusted asset M_i is completely hedged as:

$$\begin{aligned}\rho(t,i) &= 1-\exp\left(-\int_t^{t+1}\left[\Psi(\tau)-\delta(\tau,i)\right]d\tau\right) = 1-\exp\left[-\frac{\beta(t)-\varepsilon(t,i)}{\beta(t)}\int_t^{t+1}\Psi(\tau)\,d\tau\right]. \\ &= 1-\left[\alpha(\mathrm{t})\right]^{\left[\beta(t)-\varepsilon(\mathrm{t,i})\right]/\beta(t)} \qquad \forall i\in\{1,..8\}\end{aligned} \tag{11}$$

This analysis allows us to draw two important inferences as described hereafter. The trivial inequality $\Psi(\tau)>\delta(\tau,i)$ suggests that:

$$\begin{aligned}\gamma(t,i) &= \int_t^{t+1}\exp\left[-\int_t^{\tau}\delta(\tilde{\tau},i)\,d\tilde{\tau}\right]\delta(\tau,i)\,d\tau \\ &> \int_t^{t+1}\exp\left[-\int_t^{\tau}\Psi(\tilde{\tau})\,d\tilde{\tau}\right]\delta(\tau,i)\,d\tau = \varepsilon(t,i)\end{aligned}. \tag{12}$$

Essentially, because the probability $\gamma(t,i)$ considers only a single risk component and excludes any other bank exposure, it is greater than the respective likelihood $\varepsilon(t,i)$ of bank failure from a stochastic jump in a specific risk-adjusted asset when all other risky elements within the bank portfolio are still present. Moreover, we can extrapolate (interpolate) the discussion to multiple (sub) time-units and realize that the chances for a bank to remain operative in longer (shorter) time periods are set by the product (division) of the corresponding probability to evade failure within a standard time-unit. For instance, the probability for an operating bank at time *t* to remain operative throughout the time interval $(t,t+s)$ when a single risk-adjusted asset M_i is completely hedged is:

$$\begin{aligned}1-\rho(t\to t+s,i) &= \exp\left(-\int_t^{t+s}\left[\Psi(\tau)-\delta(\tau,i)\right]d\tau\right) \\ &= \prod_t^{t+s-1}\left[1-\rho(t,i)\right] = \prod_t^{t+s-1}\left[\alpha(\mathrm{t})\right]^{\left[\beta(t)-\varepsilon(\mathrm{t,i})\right]/\beta(t)} \qquad \forall i\in\{1,..8\}\end{aligned}. \tag{13}$$

We acquire the probability that an operating bank at time *t* will fail during a standard time-unit $(t,t+1)$ due to a stochastic jump in the magnitude of a specific risk-adjusted asset M_i when another single risk-adjusted asset M_j is perfectly hedged, but all other bank activities remain risky as:

$$\pi(t,i,j) = \int_t^{t+1}\exp\left(-\int_t^{\tau}\left[\Psi(\tilde{\tau})-\delta(\tilde{\tau},j)\right]d\tilde{\tau}\right)\delta(\tau,i)\,d\tau\,. \tag{14}$$

We can further develop this likelihood by generalizing equation (9) so that $\delta(\tau,i)/\left[\Psi(\tau)-\delta(\tau,j)\right]=\varepsilon(t,i)/\left[\beta(t)-\varepsilon(t,j)\right]$ and by expanding equation (11) for the risk-adjusted asset M_j as follows:

$$\begin{aligned}\pi(t,i,j) &= \frac{\delta(\tau,i)}{\Psi(\tau)-\delta(\tau,j)}\int_t^{t+1}\exp\left(-\int_t^{\tau}\left[\Psi(\tilde{\tau})-\delta(\tilde{\tau},j)\right]d\tilde{\tau}\right)\left[\Psi(\tau)-\delta(\tau,j)\right]d\tau \\ &= \frac{\varepsilon(t,i)}{\beta(t)-\varepsilon(t,j)}\left\{1-\exp\left[-\int_t^{t+1}\left[\Psi(\tau)-\delta(\tau,j)\right]d\tau\right]\right\} \\ &= \frac{\varepsilon(t,i)\,\rho(t,j)}{\beta(t)-\varepsilon(t,j)} \qquad \forall i\neq j\in\{1,..,8\}\end{aligned}. \tag{15}$$

The relation between $\pi(t,i,j)$ and $\varepsilon(t,i)$ is not deterministic. It rather depends whether the ratio $\rho(t,j)/\left[\beta(t)-\varepsilon(t,j)\right]$ is bigger than, equal to, or smaller than one. This outcome is quite intuitive. When a bank hedges a single risk-adjusted asset j, the total probability of failure is reduced, but so is the probability of failure associated with a specific risk i, since the absolute distance from the regulatory RBC ratio is now different. We also observe that $\sum_{i=1, i\neq j}^{8} \varepsilon(t,i) = \beta(t) - \varepsilon(t,j)$, therefore:

$$\sum_{\substack{i=1\\ i\neq j}}^{8} \pi(t,i,j) = \sum_{\substack{i=1\\ i\neq j}}^{8} \frac{\varepsilon(t,i)}{\beta(t) - \varepsilon(t,j)} \left\{1 - \left[\alpha(t)\right]^{\left[\beta(t)-\varepsilon(t,j)\right]/\beta(t)}\right\}. \qquad (16)$$
$$= 1 - \left[\alpha(t)\right]^{\left[\beta(t)-\varepsilon(t,j)\right]/\beta(t)} = \rho(t,j)$$

Moreover, we can adapt this outcome for economic scenarios where several risk-adjusted assets are perfectly hedged within a bank portfolio. For instance, when two risk-adjusted sources M_j and M_k with $i \neq j,k \in (1,..,8)$ can no longer endanger a bank, we nominate the probability that an operating bank at time t will fail during a standard time-unit due to a stochastic jump in the magnitude of a specific risk-adjusted asset M_i as:

$$\pi(t,i,j,k) = \frac{\varepsilon(t,i)}{\beta(t) - \varepsilon(t,j) - \varepsilon(t,k)} \left\{1 - \left[\alpha(t)\right]^{\left[\beta(t)-\varepsilon(t,j)-\varepsilon(t,k)\right]/\beta(t)}\right\} \quad \forall i \neq j,k \in (1,..,8). \qquad (17)$$

Following the same avenues described thus far, we can expand the results when three or even more risk-adjusted bank activities are completely hedged, or when there are more than eight independent risk-adjusted sources jeopardizing a particular financial institution.

4. General Estimation Guidelines

In this section we present diverse methods to evaluate the model parameters. The first suggested technique is to estimate the jump-diffusion processes of the eight risk-adjusted sources through Maximum Likelihood Estimation (MLE). Within this setting, the log-likelihood functions are:

$$l(\theta,\chi) = \sum_{t=0}^{T} \ln\left[\sum_{n=0}^{N} \frac{e^{-\lambda}\lambda^{n}}{n!} \frac{1}{\sqrt{2\pi\left(\sigma_i^2 + n\sigma_j^2\right)}} \exp\left(\frac{-\left(\chi_{t+1} - \mu_i - n\mu_j\right)^2}{2\left(\sigma_i^2 + n\sigma_j^2\right)}\right)\right], \qquad (18)$$

where the notations correspond to equation (1), (2), and (3). Yet, due to the non-linearity of this type of MLE, we ought to compute the unobservable parameter θ numerically. Craine, Lochstoer, and Syrtveit (2000) provide further details on the complexity of this numerical estimation method.

In addition, we recommend a second straightforward technique to estimate the probability $\delta(\tau,i)$ for a single instantaneous stochastic jump that could independently cause a bank failure at a specific time. We can assess the explicit intensity Poisson process λ_i within the GBM by measuring the necessary percentage changes in the magnitudes of each risk-adjusted asset M_i, $i \in (1,..,8)$, that could immediately cause a bank failure, and count the number of such events in the previous years. The corresponding Poisson mean λ_i is therefore the number of prior jumps divided by the number of standard time-units measured. In this case, we can apply the rough approximation $\delta(\tau,i) = e^{-\lambda_i}\lambda_i$ for a single stochastic jump. When there are no prior observable jumps with the necessary scale for a bank failure, we suggest to assigning a nominal value, for instance $\lambda_i = 0.001$.[13]

Alternatively, we advise a third technique that deploys Monte Carlo simulations of the

13 A similar method is often undertaken within the credit ratings transition matrix to settle low default probabilities of highly rated bonds. For example, Kijima and Komoribayashi (1998) propose to appoint a minimal probability for the 'AAA' scores to instantaneously migrate to the absorbing state of default as a necessary correction to the standard Markov chain model offered by Jarrow, Lando, and Turnbull (1997).

stochastic processes by replicating the number of observable jumps, identifying the jump times, and reproducing the GBM on the time intervals between consecutive jumps. This common technique is likely to yield more accurate predictions, yet it requires a large volume of prior records to accurately calibrate the simulations. A fourth estimation method advocates further associating the stochastic behaviors of the risk-adjusted bank sources to different macroeconomic factors. This approach requires a slight modification of equation (1) to incorporate the relevant economic cycle for example.

All together, we advocate that bank examiners, whenever feasible, should implement more than one of the recommended methods to evaluate the stochastic processes of the risky bank assets. These estimators serve as predictors to future Poisson jumps, and better assessments should yield higher model accuracy. Once we obtain the independent instantaneous probabilities $\delta_{\tau,i}$, we can aggregate them in equation (4) and set the constant relative risk proportion in equation (5). These parameters are sufficient to construct the whole model.

5. Empirical Illustration of the Model

We now demonstrate the model with some genuine examples taken from the Bank Regulatory database. This file, offered by the Federal Reserve Bank of Chicago, contains quarterly balance sheet and income statement records, off-balance sheet items, and risk based capital measures for numerous regulated U.S. commercial banks, from January 1976 to December 2008. Nonetheless, RBC was not habitually reported before 1991.

Within the merged database we are able to identify 12,577 banks having 147,720 observations with non-negative records of Tier 1 and Tier 2 capitals, book value of risk-weighted assets under Basel RBC guidelines, market value of risky assets held in trading accounts, and all relevant ingredients of commercial, real estate, and industrial loans (risk-adjusted source 1), gross loans and leases (risk-adjusted source 2), agricultural loans (risk-adjusted source 3), income earned but not collected on loans (risk-adjusted source 4), loans to individuals (risk-adjusted source 5), all other loans (risk-adjusted source 7), and total investment securities (risk-adjusted source 8). Unfortunately, we are unable to clearly categorize banks' exposure to foreign assets within this dataset, thus we exclude from the analysis risk-adjusted source 6, as identified by the OCC's handbook (2001).[14]

We compute total risk-based capital as the sum of common stock equity, perpetual preferred stock and related surplus, other surplus, undivided profits and capital reserves, cumulative foreign currency translation adjustments, but less net unrealized loss on marketable equity securities. We further estimate risk-adjusted total assets as the sum of stripped mortgage backed securities, residual and subordinated asset backed securities, industrial development bonds, debt to private obligors, margin accounts on future contracts, net deferred tax assets, other real estate owned, differences between the fair value and the amortized cost of the banks' available-for-sale debt securities, and special risk-weighted assets under the classification of the Basel RBC guidelines.

After eliminating instances with missing records we are now left with 4,420 banks having 30,755 observations. From this pool, we cluster two groups of banks. The first group contains only banks that have failed according to the universal definition, explicitly banks with RBC adequacy ratios below eight percent, but only those banks with at least eight consecutive quarters of prior records while conducting normal operations.[15] The second control group is comprised of non-failed banks with enough prior records, but we intentionally select the same number of banks and only those banks with similar financial characteristics as in the first group. We therefore match the two sub-samples with respect to the banks' risk-adjusted total assets and the approximate time period.

We wish to explore the functionality of the proposed model with both failed and non-failed banks that have enough precedent documentation to demonstrate the differences between these two

[14] The OCC list of eight bank typical activities does not align perfectly with the Federal Reserve Bank of Chicago record of risk based capital measures. Nonetheless, despite a minor discrepancy in names, these two catalogs convey parallel objectives.

[15] The database offers a special indicator on whether banks' total capital remains above eight percent of the risk-adjusted total assets, yet this record is incomplete. Thus, we compute the tangible RBC ratios directly from the sample and use the indicators whenever available merely to authenticate our inferences.

groups. We also aim to assemble a solid view of how banks manage their risk-adjusted sources on the road to failure. We deliberately focus on the two years prior to failures not only to track how banks administer their risky assets during the critical time period of distress, but also to preserve enough examples for this analysis. Within the first group we ultimately isolate 29 failed banks having altogether 354 successive quarterly observations. In the control group we gather 29 non-failed banks having altogether 355 successive quarterly observations. In both sub-samples we obtain an average of slightly more than twelve consecutive quarters of a year per bank. We therefore assess the pertinent model probabilities as forward-looking out-of-sample measurements, based on accessible data within rolling windows at each point-in-time.

We are able to form two highly comparable groups with equal sizes and equal eras. 24 of the control non-failed banks have their risk-adjusted total assets within plus or minus five percent of the matched failed banks' corresponding figures. Four of the non-failed banks have their risk-adjusted total assets within plus or minus ten percent of the parallel failed banks' respective values, and only one non-failed bank has its risk-adjusted total assets within plus or minus 15 percent of the failed bank's analogous quantity.

Table 1 demonstrates that all of the failed banks breached the minimum required RBC adequacy ratio of eight percent from 1992 to 1993, hence during the savings and loan crisis of the early 1990s. Table 2 examines an equivalent time period for the non-failed banks. These final sub-samples utilize a little more than three years of data per bank. Therefore, these sub-samples are sufficient to approximate the intensity of prior jumps in the risk-adjusted bank assets, yet they are rather inadequate to comprehend a more accurate stochastic behavior of these jumps from Monte Carlo simulations. Furthermore, since both sub-samples are drawn from the same relatively-short time frame, from 1990 to 1993, we do not incorporate macroeconomic variables into the stochastic processes of the risky bank assets.

From Table 1 we learn that commercial real estate and construction lending (risk source 1), leasing finance (risk source 2), retail credit (risk source 5), and investment securities and derivatives (risk source 8) are the most prevalent exposures within the two years prior to banks' failures. In contrast, agricultural lending (risk source 3), accounts receivable and inventory financing (risk source 4), and other loans purchased at discount (risk source 7) are not a significant source of risk for distressed banks during the savings and loan crisis of the early 1990s. Table 2 presents parallel results among the non-failed banks. Therefore, we must take further computations to distinguish between ultimately failed and non-failed commercial banks.

Table 1 provides more information on the mean and the maximum values of the RBC ratios among the 29 failed banks during the two years prior to failures, as well as the minimum recorded RBC ratios at the corresponding quarters of failures. We observe that the vast majority of failed banks maintain RBC ratios that are marginally above the minimum required by law. These common patterns however, are consistent among the non-failed banks in Table 2. Most of the minimum, mean, and maximum RBC ratios of the non-failed banks are only slightly higher than the regulatory threshold.

An independent equal-sizes two-sample t-test on the mean values suggests a statistically significant difference between the two sub-samples.[16] However, we must reject the economic significance at the individual firm's level. About half of the measured RBC ratios of the non-failed banks are below ten percent, and about two thirds of the measured RBC ratios of the non-failed banks are below 11 percent. We find comparable proportions within the first group of ultimately failed banks in the two years prior to failures. This comparison further motivates our endeavor to expose bank credit risk by disentangling the probabilities associated with the different risk-adjusted bank sources.

[16] Explicitly, $\bar{X}_1 = 0.094$, $\bar{X}_2 = 0.119$, $\sigma_1 = 0.012$, $\sigma_2 = 0.036$, $S_{1,2} = 0.027$, and $t - Value = 3.506$, which is statistically significant at the 0.01 level, or better.

Table 1
Descriptive Statistics for the 29 Failed Banks

The table below presents characteristics of the first sub-sample of failed banks sorted by ID numbers. Corresponding RBC ratios show the minimum value (at the failure quarter), and the mean figure and the maximum quantity during the two years prior to failure. Throughout these two years we count the number of quarters with positive bank exposure to the various risk-adjusted sources. We average these numbers at the bottom as a raw measure of exposure to individual risk components.

Bank ID Number	Failure Quarter	Min RBC Ratio	Mean RBC Ratio	Max RBC Ratio	Number of Quarters During the Two Years before the Actual Bank Failure with Exposure to…						
					Risk 1	Risk 2	Risk 3	Risk 4	Risk 5	Risk 7	Risk 8
52764	1993/Q4	0.056	0.094	0.115	8	8	0	4	8	0	8
64619	1993/Q4	0.078	0.130	0.170	8	8	0	0	4	0	8
236340	1993/Q1	0.079	0.090	0.096	8	8	8	8	8	0	8
324902	1993/Q2	0.073	0.094	0.110	8	8	0	0	0	0	8
332505	1992/Q4	0.071	0.099	0.109	8	8	0	3	8	3	8
337340	1993/Q4	0.079	0.090	0.094	8	8	0	1	8	0	8
351122	1993/Q1	0.080	0.087	0.095	8	8	0	3	8	2	8
368447	1993/Q3	0.080	0.083	0.092	8	8	0	3	8	3	8
437521	1993/Q1	0.080	0.087	0.092	8	8	0	2	8	0	8
455253	1992/Q4	0.079	0.102	0.137	4	8	0	3	8	0	7
483425	1993/Q2	0.079	0.089	0.111	8	8	0	3	8	4	8
517768	1993/Q1	0.076	0.087	0.094	8	8	8	3	8	0	6
636007	1993/Q4	0.078	0.106	0.122	8	8	0	0	8	0	8
675332	1992/Q4	0.080	0.087	0.096	8	8	0	2	8	8	8
710532	1993/Q2	0.069	0.089	0.097	8	8	0	3	8	1	8
800965	1992/Q4	0.078	0.087	0.092	8	8	0	6	8	8	8
810106	1993/Q2	0.078	0.097	0.105	8	8	0	0	8	1	8
863915	1993/Q3	0.076	0.087	0.092	8	8	2	1	8	1	8
868769	1993/Q3	0.073	0.084	0.087	8	8	2	3	8	8	8
888534	1993/Q4	0.078	0.089	0.101	8	8	0	2	8	8	8
928449	1992/Q4	0.077	0.085	0.089	8	8	8	4	8	2	8
928869	1993/Q4	0.075	0.084	0.090	8	8	0	4	8	7	8
935148	1993/Q4	0.076	0.118	0.147	8	8	0	0	8	0	8
938653	1993/Q1	0.077	0.092	0.104	8	8	0	2	8	0	5
958204	1992/Q4	0.078	0.090	0.098	8	8	0	4	8	0	8
984623	1993/Q3	0.078	0.084	0.087	8	8	0	4	8	1	8
1216022	1993/Q4	0.063	0.122	0.229	0	8	0	0	8	0	0
1412758	1993/Q1	0.072	0.107	0.131	0	8	0	0	8	0	8
1434826	1993/Q4	0.079	0.091	0.103	8	8	0	1	8	0	8
Average:		0.076	0.094	0.110	7.31	8.00	0.97	2.38	7.59	1.97	7.52

We then add up the number of actual variations with at least the same magnitudes as the necessary percent changes for bank failures in the prior years, and divide these figures by the respective ten-day intervals. This gives us the Poisson intensities λ_i, $i \in 1,..,8$, $i \neq 6$, and the rough approximations for the instantaneous probabilities $\delta\ \tau, i = e^{-\lambda_i}\lambda_i$. We further estimate the remaining likelihoods directly from the model derivations.

Table 3 summarizes how the average failure probabilities of the 29 examined banks adjust throughout the two years prior to the actual failures. It also displays the average number of risk-adjusted sources as a raw measure of exposure across the 29 distressed banks. We find that failed banks generally neither increase nor decrease their exposure to more or less risk-adjusted lending activities in their troubled periods of time. Instead, failed banks regularly maintain a relatively steady exposure to slightly more than four key threats (risk-adjusted sources 1, 2, 5, and 8). Nevertheless, both the instantaneous probabilities of bank failure and the supplementary default likelihoods progress through time. These variables serve as the fundamental indicators that clearly distinguish expected failed banks from lending institutions that continue normal operations.

Table 2

Descriptive Statistics for the 29 Non-Failed Banks with Similar Characteristics

The table below presents characteristics of the second sub-sample of non-failed banks sorted by ID numbers. Corresponding RBC ratios show the minimum value, the mean figure, and the maximum quantity during the two years prior to failure. Throughout these two years we count the number of quarters with positive bank exposure to the various risk-adjusted sources. We average these numbers at the bottom as a raw measure of exposure to individual risk components.

Bank ID Number	Last Examined Quarter	Min RBC Ratio	Mean RBC Ratio	Max RBC Ratio	Number of Quarters During the Two Years before the Last Examined Quarter with Exposure to…						
					Risk 1	Risk 2	Risk 3	Risk 4	Risk 5	Risk 7	Risk 8
2648	1993/Q4	0.081	0.088	0.097	8	8	0	1	8	2	8
9807	1993/Q4	0.091	0.111	0.132	8	8	0	0	3	0	8
10746	1993/Q4	0.081	0.093	0.114	8	8	0	0	8	0	8
19534	1993/Q4	0.123	0.146	0.162	8	8	0	0	8	0	8
26916	1993/Q4	0.108	0.123	0.155	8	8	0	0	8	0	8
28303	1993/Q4	0.097	0.101	0.109	8	8	0	0	4	0	8
30418	1993/Q4	0.097	0.128	0.155	0	0	0	0	0	0	8
40305	1993/Q4	0.085	0.094	0.107	8	8	0	0	8	2	8
45122	1993/Q4	0.098	0.105	0.113	8	8	0	0	8	2	8
46549	1992/Q4	0.125	0.137	0.146	8	8	0	0	0	0	8
54218	1993/Q2	0.081	0.093	0.100	8	8	0	0	8	2	8
61476	1993/Q2	0.147	0.172	0.201	8	8	0	0	0	0	8
66613	1993/Q4	0.138	0.174	0.261	0	8	0	0	8	0	0
67311	1993/Q4	0.093	0.102	0.111	8	8	0	0	8	0	8
116938	1993/Q4	0.087	0.098	0.111	8	8	0	2	8	0	8
225308	1993/Q4	0.087	0.103	0.116	8	8	0	1	8	0	8
236340	1993/Q4	0.080	0.088	0.096	8	8	8	8	8	1	8
486752	1993/Q4	0.102	0.159	0.222	8	8	0	0	8	0	0
500050	1993/Q4	0.086	0.092	0.101	8	8	0	0	8	0	8
587800	1993/Q4	0.125	0.136	0.148	8	8	0	0	8	0	8
588946	1993/Q4	0.084	0.091	0.102	8	8	0	0	8	1	8
601546	1993/Q4	0.080	0.089	0.098	8	8	0	2	8	6	8
825007	1993/Q2	0.080	0.089	0.109	8	8	0	4	8	2	7
862338	1993/Q4	0.085	0.099	0.111	8	8	0	0	5	0	8
930563	1993/Q4	0.094	0.101	0.107	8	8	0	0	0	0	8
1000445	1993/Q3	0.084	0.093	0.102	8	8	0	0	8	0	8
1029325	1993/Q4	0.080	0.206	0.337	8	8	0	0	0	0	1
1228623	1993/Q4	0.167	0.225	0.274	8	8	0	0	5	0	0
880350	1993/Q4	0.099	0.114	0.134	8	8	0	0	8	0	8
Average:		0.099	0.119	0.142	7.45	7.71	0.28	0.62	6.10	0.62	6.90

Following our second proposed estimation technique, across both sub-samples we measure the absolute distance to failure as the current RBC less eight percent of the risk-adjusted total assets. We further calculate the required percentage change for each risk-adjusted source that can cause an immediate bank failure, while classifying the seven observable risky assets as either "hedged" or "exposed." Whenever a momentary exposure to a specific risky asset is lower than the existing distance to failure, we categorize this risky source as "hedged." Otherwise, we grade it as "exposed." For each failed (non-failed) bank we aggregate the number of quarters during the two years before the actual failure time (the last examined quarter) with positive exposures to the various risk sources and average these figures in Table 1 and Table 2.

Table 3
Empirical Findings among the 29 Failed Banks

The table below demonstrates how the average number of risk-adjusted sources with positive exposure and the average failure probabilities as derived from the model progress throughout the two years prior to the actual bank failures across the 29 failed banks.

Average Values	Failure - 8 Qtrs	Failure - 7 Qtrs	Failure - 6 Qtrs	Failure - 5 Qtrs	Failure - 4 Qtrs	Failure - 3 Qtrs	Failure - 2 Qtrs	Failure - 1 Qtr
# Risk - Adjusted Sources	4.448	4.379	4.345	4.345	4.586	4.517	4.552	4.552
$\delta(\tau,1)$	0.009	0.021	0.027	0.029	0.031	0.031	0.034	0.038
$\delta(\tau,2)$	0.015	0.028	0.036	0.036	0.038	0.040	0.042	0.045
$\delta(\tau,3)$	0.003	0.004	0.005	0.004	0.003	0.004	0.004	0.004
$\delta(\tau,4)$	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.003
$\delta(\tau,5)$	0.008	0.012	0.013	0.016	0.016	0.019	0.022	0.023
$\delta(\tau,7)$	0.002	0.005	0.003	0.003	0.006	0.005	0.004	0.007
$\delta(\tau,8)$	0.001	0.001	0.001	0.001	0.003	0.003	0.003	0.003
$\Psi(\tau)$	0.039	0.073	0.086	0.089	0.099	0.103	0.111	0.122
$\eta(t,1)$	0.171	0.242	0.278	0.300	0.309	0.306	0.335	0.336
$\eta(t,2)$	0.273	0.324	0.377	0.387	0.373	0.377	0.396	0.389
$\eta(t,3)$	0.105	0.073	0.070	0.047	0.035	0.038	0.030	0.024
$\eta(t,4)$	0.094	0.048	0.034	0.031	0.031	0.031	0.017	0.021
$\eta(t,5)$	0.155	0.169	0.139	0.148	0.151	0.165	0.165	0.156
$\eta(t,7)$	0.110	0.097	0.067	0.055	0.064	0.050	0.030	0.049
$\eta(t,8)$	0.094	0.048	0.034	0.031	0.037	0.033	0.027	0.026
$\alpha(t)$	0.755	0.571	0.510	0.495	0.446	0.438	0.413	0.378
$\beta(t)$	0.245	0.429	0.490	0.505	0.554	0.562	0.587	0.622
$\varepsilon(t,1)$	0.001	0.018	N/A	N/A	N/A	N/A	0.054	N/A
$\varepsilon(t,2)$	0.001	0.044	N/A	N/A	N/A	N/A	0.054	N/A
$\varepsilon(t,3)$	0.001	0.011	N/A	N/A	N/A	N/A	0.001	N/A
$\varepsilon(t,4)$	0.001	0.001	N/A	N/A	N/A	N/A	0.001	N/A
$\varepsilon(t,5)$	0.001	0.001	N/A	N/A	N/A	N/A	0.014	N/A
$\varepsilon(t,7)$	0.001	0.028	N/A	N/A	N/A	N/A	0.014	N/A
$\varepsilon(t,8)$	0.001	0.001	N/A	N/A	N/A	N/A	0.001	N/A
$\rho(t,3)$	0.019	0.102	0.102	0.105	0.118	0.140	0.199	0.173
$\rho(t,7)$	0.102	0.133	0.125	0.139	0.134	0.140	0.125	0.184
$\pi(t,1,3)$	0.001	0.047	0.040	0.034	0.037	0.033	0.058	0.037
$\pi(t,1,7)$	0.047	0.031	0.001	0.018	0.034	0.039	0.012	0.042

We realize that except for the exposure to agricultural lending (risk source 3), all other bank instantaneous failure probabilities $\delta(\tau,i)$, $i \in \{1,2,4,5,7,8\}$ increase over time. As expected, these likelihoods are relatively small, since they convey the immediate chances for a bank failure from individual origins. Nevertheless, the entire instant bank failure probability $\Psi(\tau)$ is robust and almost monotonically increasing on the way to breach the minimum required RBC ratio. From a 3.9% immediate failure chances eight quarters before the actual failure, failed banks exhibit a 12.2%

instantaneous probability of default in the last quarter of operation. This parameter can serve both internal and external bank examiners as a red flag when testing the survivability of commercial banks.

Because commercial real estate and construction lending (risk-adjusted source 1), leasing finance (risk-adjusted source 2), and retail credit (risk-adjusted source 5) experience the larger expansions in instantaneous failure probabilities during the complete time frame of two years prior to the actual bank failures, their relative risk proportions $\eta(t,i)$, $i \in \{1,2,5\}$ surpass all other corresponding figures.

The probabilities that an operating bank will continue to be operative within the next ten days $\alpha(t)$ are radically reduced within these two years from 75.5% initially to 37.8% at the end. Consequently, the complement likelihoods $\beta(t)$ for a bank failure are considerably amplified throughout this time frame, from 24.5% at the beginning to 62.2% just before the failure. These quantities perfectly reside within the acceptable range of corporate default likelihoods during distressed periods, thus further validate our model.[17]

Since the final sample does not include any record of bank exposure to a single risk-adjusted asset, where all other risk sources are perfectly hedged, all the probabilities $\gamma(t,i)$, $i \in \{1,2,3,4,5,7,8\}$ are not applicable in this analysis. The probabilities $\varepsilon(t,i)$ are measured only when all seven observable risk-adjusted assets are not hedged, thus they lack observations as well, but they do appear within the eight, seven, and two quarters of a year prior to bank failures. As predicted by the model, these likelihoods are relatively low, but overall, they incorporate a general tendency to increase over time.[18] Following the identifiable dominance of commercial real estate and construction lending (risk-adjusted source 1) and leasing finance (risk-adjusted source 2), the corresponding likelihoods $\varepsilon(t,1)$ and $\varepsilon(t,2)$ exhibit the higher increments over time.

The final sub-sample of failed banks dictates that only agricultural lending (risk source 3) and other loans purchased at discount (risk source 7) are perfectly hedged when all the other six apparent risk-adjusted assets are present. Therefore, we are able to compute only $\rho(t,3)$ and $\rho(t,7)$. Both have a tendency to increase over time.

To depict a more comprehensive view, we further demonstrate the computations of $\pi(t,1,3)$ and $\pi(t,1,7)$ as the bank failure probabilities within the next ten days due to a stochastic jump in the magnitude of commercial real estate and construction lending (risk source 1) when either agricultural lending (risk source 3) or other loans purchased at discount (risk source 7) are perfectly hedged, respectively. These odds express minor fluctuations through time, while continuously capturing respective portions out of $\rho(t,3)$ and $\rho(t,7)$, as illustrated in equation (16). Furthermore, we are able to demonstrate the inconclusive relation between $\pi(t,i,j)$ and $\varepsilon(t,i)$, as expressed in equation (15). When we compare $\pi(t,1,3)$ and $\pi(t,1,7)$ to $\varepsilon(t,1)$ we observe that the latter measurement is bigger than, equal to, or smaller than the first two probabilities, which involve the hedging of agricultural lending or other loans purchased at discount, respectively.

In contrast, Table 4 portrays a different image for the non-failed banks. Similar to the first group of failed banks, the average number of risk-adjusted assets with positive exposure among the non-

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[17] Vassalou and Xing (2004) process numerous corporate default probabilities by using the Merton (1974) structural model.

[18] Since the final sub-sample of failed banks does not offer valid observations to calculate the probabilities $\gamma(t,i)$, we cannot contrast them with the likelihoods $\varepsilon(t,i)$ as suggested in equation (12), yet the fairly low values of the latter measurements suggest that these probabilities are bounded from above.

failed banks is relatively stable throughout the two years under investigation. Although the arithmetic mean among the non-failed banks approaches four from below, the difference between the two groups is not economically significant for discrete banks. Similar to the first group of

Table 4
Empirical Findings among the 29 Non-Failed Banks with Similar Characteristics

The table below demonstrates how the average number of risk-adjusted sources with positive exposure and the average failure probabilities as derived from the model progress throughout the two years prior to the last examined quarter of the 29 non-failed banks.

Average Values	Success - 8 Qtrs	Success - 7 Qtrs	Success - 6 Qtrs	Success - 5 Qtrs	Success - 4 Qtrs	Success - 3 Qtrs	Success - 2 Qtrs	Success - 1 Qtr
# Risk - Adjusted Sources	3.893	3.893	3.679	3.643	3.714	3.786	3.536	3.464
$\delta(\tau,1)$	0.019	0.023	0.024	0.024	0.025	0.022	0.024	0.023
$\delta(\tau,2)$	0.021	0.021	0.023	0.026	0.024	0.023	0.023	0.024
$\delta(\tau,3)$	0.002	0.002	0.003	0.002	0.003	0.001	0.003	0.003
$\delta(\tau,4)$	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.003
$\delta(\tau,5)$	0.013	0.011	0.011	0.011	0.014	0.011	0.011	0.012
$\delta(\tau,7)$	0.002	0.004	0.002	0.002	0.002	0.003	0.001	0.001
$\delta(\tau,8)$	0.025	0.022	0.026	0.024	0.029	0.030	0.029	0.028
$\Psi(\tau)$	0.083	0.085	0.089	0.090	0.097	0.092	0.093	0.093
$\eta(t,1)$	0.198	0.262	0.256	0.261	0.247	0.236	0.238	0.233
$\eta(t,2)$	0.196	0.206	0.223	0.255	0.226	0.242	0.248	0.249
$\eta(t,3)$	0.059	0.046	0.046	0.043	0.029	0.027	0.030	0.028
$\eta(t,4)$	0.049	0.036	0.037	0.033	0.026	0.032	0.025	0.028
$\eta(t,5)$	0.130	0.103	0.121	0.122	0.138	0.100	0.109	0.126
$\eta(t,7)$	0.058	0.057	0.041	0.039	0.029	0.037	0.027	0.022
$\eta(t,8)$	0.309	0.289	0.275	0.248	0.305	0.326	0.323	0.313
$\alpha(t)$	0.555	0.525	0.511	0.504	0.464	0.494	0.472	0.480
$\beta(t)$	0.445	0.475	0.489	0.496	0.536	0.506	0.528	0.520
$\gamma(t,8)$	0.001	0.001	0.001	0.001	0.045	0.051	0.055	0.058
$\varepsilon(t,3)$	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
$\rho(t,3)$	0.133	0.166	N/A	N/A	N/A	0.190	N/A	N/A
$\rho(t,7)$	0.139	0.118	0.154	0.125	0.225	N/A	0.204	0.254
$\pi(t,1,3)$	0.031	0.023	N/A	N/A	N/A	0.016	N/A	N/A
$\pi(t,1,7)$	0.018	0.016	0.026	0.012	0.036	N/A	0.038	0.045

ultimately failed banks, non-failed banks are ordinarily exposed to risk-adjusted sources 1, 2, 5, and 8, but the instantaneous probabilities of bank failure $\delta\ \tau,i$ and $\Psi\ \tau$ as well as the accompanying default likelihoods $\eta\ t,i$ do not exhibit a particular tendency to rise over time. This disparity is robust and abundant across the comparable banks therefore economically significant at both the means' level and at the individual firms' level.

Moreover, the probabilities that an operating bank will continue to be operative within the next ten days $\alpha\ t$ are moderately changed within these two years from 55.5% initially to 48.0% at the last examined quarter.[19] As a result, the complement likelihoods $\beta\ t$ for a bank failure are reasonably adjusted throughout this time frame, from 44.5% at the beginning to 52.0% at the end. Accordingly, the remaining failure probabilities $\varepsilon\ t,3$, $\rho\ t,3$, $\rho\ t,7$, $\pi\ t,1,3$ and $\pi\ t,1,7$ do not show any clear propensity. Finally, the sub-sample of non-failed banks contains only one occasion where the likelihood $\gamma\ t,8$ is attainable, and although we can observe a minor tendency to rise over time, we cannot draw meaningful economic inferences from this single occurrence.

6. Summary

In this study we develop a notional model that assesses bank credit risk. We first isolate eight independent competing risks for typical banking institutions. We then measure the likelihoods for conjectural jumps in commercial real estate and construction lending, leasing finance, agricultural lending, accounts receivable and inventory financing, retail credit, foreign assets, other loans purchased at discount, and investment securities and derivatives. Next, we classify these modules as temporarily "hedged" or "exposed," and then evaluate the instantaneous failure probabilities attached to each risk component by analyzing its precedent stochastic behavior. We further use these inferences to compose the complete risk profile of a typical commercial bank and to derive numerous failure probabilities, depending on the pertinent economic setting: when the underlying bank is fully exposed or partially hedged to these competing risks.

We suggest several estimation techniques for the model parameters. Each has its own advantages and disadvantages. While the first analytical approach aims to be the most accurate, it is also the most difficult to implement. The second proposed method is straightforward, but it only gives rough approximations for the instantaneous failure probabilities from comparable past events. The third procedure incorporates Monte Carlo simulations that should improve the model accuracy, yet it requires sufficient data for precise calibration of the simulations. The fourth alternative embeds macroeconomic variables into the stochastic behavior of the risk-adjusted bank assets. This practice is likely to be more applicable for diverse economic settings, but its contribution is somewhat elusive during short and steady economic cycles.

We further validate the theoretical scheme by contrasting data on two groups of ultimately failed and non-failed U.S. banks from 1990 to 1993. From the Bank Regulatory database we form two highly comparable groups with 29 commercial banks in each collection. We match these sub-samples based on the time period under investigation and the risk-adjusted total assets of the individual banks within. We then evaluate the particular risk characteristics of the specific banking institutions and their mean levels.

We find little evidence for differences in the minimum, mean, and maximum values of the inclusive RBC ratios between the ultimately failed and the non-failed banks. These dissimilarities however, are not economically significant at the individual firm's level. We also realize that failed and non-failed banks have similar properties with respect to the number of consecutive quarters during the two years prior to the actual failure, or the last examined quarter for the non-failed banks, with positive exposure to the various risk modules. It appears that throughout the savings and loan crisis of the early 1990s, both failed and non-failed banks were mostly exposed to commercial real estate and construction lending, leasing finance, retail credit, and investment securities and derivatives.

In contrast, we discover economically significant differences between these two groups with respect to the instantaneous failure probabilities and the complete risk profiles as derived from the theoretical model. Furthermore, we detect meaningful information concerning the credit quality of banks not only in the absolute quantities of failure probabilities but also within their evolution over

[19] We recall that these probabilities are explicitly estimated for a particular set of 29 non-failed banks having similar distressed symptoms as the first group of 29 eventually failed banks. These quantities merely serve as a model illustration and are not applicable for a general sample of non-failed banks.

time. The discrepancies in the absolute values and the tendencies are robust both at the aggregate and the individual firm levels. These findings authenticate the superiority of the proposed model over naïve approaches that examine merely RBC ratios. We therefore recommend policy makers as well as internal and external bank examiners to deploy the present theory while evaluating bank credit risk.

The Prompt Corrective Action (PCA) is a U.S. Federal law from 1991, which sets regulatory minimum thresholds for bank capital and further defines progressive penalties against banks that exhibit gradually deteriorating capital ratios. The greatest contribution of the proposed model rests with its ability to detect worsening tendencies in the various components of the RBC ratios well before a forced failure is required by the FDIC. We therefore deduce that the current analytical scheme further follows the spirit of the law; it allows banks to address risk problems while they are still controllable.

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Appendix 1

Notation	Meaning
$i \in \; 1,..,8$	Identifier for the eight risk-adjusted assets within typical banks' common activities
M_i	The sovereign magnitude of the eight risk-adjusted assets
μ	The drift of the underlying stochastic process of a specific sovereign magnitude
σ	The diffusion of the underlying stochastic process of the specific sovereign magnitude
W	A standard Wiener process
dJ	A Poisson counter for the jump process
λ	The mean of the jump Poisson process
ω	A draw from a Normal distribution
χ	The periodic logarithmic changes for the eight independent magnitudes
υ	A draw from a Geometric Brownian Motion
n	The number of jumps being modeled
δ	The probability for a single instantaneous stochastic jump that could trigger a bank failure
τ	Specific time within the interval *(t, t+1)*, where ten days are set as a standard time-unit
t	General time that defines the interval *(t, t+1)*, where ten days are set as a standard time-unit
Δ	The length of the standard time interval *(t, t+1)*
s	A counter for standard time-units to define a longer time interval *(t, t+s)*
Ψ	Total bank failure probability as the sum of the disjoint instantaneous probabilities for stochastic jumps
$\eta \;\; t,i$	A constant relative risk proportion for risk-adjusted asset i at a specific time τ
$\alpha \;\; t$	The probability that an operating bank at time *t* will continue to be operative until *t+10* days
$\beta \;\; t$	The probability that an operating bank at time *t* will fail before *t+10* days without identifying any specific stochastic jump causing this failure
$\varepsilon \;\; t,i$	The probability that an operating bank at time *t* will fail before *t+10* days due to a stochastic jump in the magnitude of a specific risk-adjusted asset M_i when all other risk-adjusted assets are not hedged
$\gamma \;\; t,i$	The probability that an operating bank at time *t* will fail before *t+10* days due to a stochastic jump in the magnitude of a specific risk-adjusted asset M_i when all other risky assets are perfectly hedged
$\rho \;\; t,i$	The probability that an operating bank at time *t* will fail before *t+10* days when a specific risk-adjusted asset M_i is perfectly hedged
$\pi \;\; t,i,j$	The probability that an operating bank at time *t* will fail before *t+10* days due to a stochastic jump in the magnitude of a specific risk-adjusted asset M_i when a single risk-adjusted asset M_j is perfectly hedged
$\pi \;\; t,i,j,k$	The probability that an operating bank at time *t* will fail before *t+10* days due to a stochastic jump in the magnitude of a specific risk-adjusted asset M_i when two risk-adjusted assets M_j and M_k are perfectly hedged
$f_i \;\; \tau$	A density function for the bank's operative life span with a single risk-adjusted source
$F_i \;\; \tau$	A cumulative distribution function for the bank's operative life span with one risky asset
θ	General parameter of the maximum likelihood estimation

Appendix 2

For each risk-adjusted asset $i \in \{1,..,8\}$ we designate $f_i(\tau)$ and $F_i(\tau)$ as the Probability Density Function (PDF) and the Cumulative Distribution Function (CDF) of the bank's expected operative life span if the sovereign magnitude M_i acts alone, respectively. In addition, we delineate the hazard function:

$$\delta(\tau, i) = -\frac{d}{d\tau}\ln\left[1 - F_i(\tau)\right] = \frac{f_i(\tau)}{1 - F_i(\tau)} \qquad 0 \le \tau < \infty . \tag{A2.1}$$

In our context, a bank remains fully operative as long as no stochastic jumps occur among all sovereign magnitudes M_i, thus a bank's existence has a CDF $F(\tau)$ as:

$$F(\tau) = 1 - \prod_{i=1}^{8}\left[1 - F_i(\tau)\right]. \tag{A2.2}$$

In this case, we obtain the total probability for a bank failure as:

$$\Psi(\tau) = -\frac{d}{d\tau}\ln\left[1 - F(\tau)\right] = \sum_{i=1}^{8}\delta(\tau, i) . \tag{A2.3}$$

We rewrite the assumption for a constant relative proportion in equation (5) as $\delta(\tau, i) = \eta(t, i)\Psi(\tau)$, and by using (A1) and (A3) we can integrate both sides of this equality from t to $\tau \in (t, t+1)$ as:

$$\ln\left[\frac{1 - F_i(\tau)}{1 - F_i(t)}\right] = \eta(t, i)\ln\left[\frac{1 - F(\tau)}{1 - F(t)}\right], \tag{A2.4}$$

which confirms that for every $\tau \in (t, t+1)$ the quantity

$$\left[\frac{1 - F_i(\tau)}{1 - F_i(t)}\right]^{1/\eta(t,i)} \tag{A2.5}$$

is independent of any specific risk-adjusted asset i.

Once the assumption for a constant relative proportion in equation (5) holds for the complete feasible time domain $(0, \infty)$, we can set $t = 0$ so that $F_i(0) = 0$, $\eta(t, i) = \eta(i)$, and rewrite (A5) as:

$$\left[1 - F_i(\tau)\right]^{\eta(j)/\eta(i)} = 1 - F_j(\tau) \qquad \forall j \in \{1,..,8\}, j \ne i . \tag{A2.6}$$

Therefore, $F_j(\tau)$ is the CDF of the minimum of the independent ratios $\eta(j)/\eta(i)$, each with a CDF $F_i(\tau)$. Moreover, (A2.6) is defined for every positive value of $\eta(j)/\eta(i)$, and in particular it can be satisfied by the properties of distributions of minimum random variables, including the Weibull distribution and the more specific Exponential distribution. For example, the CDF $F_i(\tau) = 1 - \exp\left[-\frac{\tau}{\eta(j)/\eta(i)}\right], \forall i \in \{1,..,8\}$ abides equation (A2.6). Therefore, these two distributions may serve as a sufficient condition and a prudent bridge between the stochastic behavior of the risk-adjusted bank assets in equations (1) to (5), and the remaining theory as outlined in equations (6) through (17) in the main text.