Credit Risk Analysis of Credit Card Portfolios under Economic Stress Conditions

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We develop an empirical framework for analyzing the credit risk in a generic portfolio of revolving credit accounts and apply it to a representative panel data set of credit card accounts. These data cover the period of the most recent recession, providing the opportunity to analyze the portfolio's performance under conditions of significant economic stress. We consider a traditional framework in which expected loss is represented in terms of probability of default (PD), loss given default (LGD), and exposure at default (EAD). The unsecured and revolving nature of credit card lending is modeled in this framework. Our results indicate that the level and change in unemployment play a significant role in the probability of transition across delinquency states in general and PD in particular. The effect is heterogeneous and proportionally larger for high-credit-score and high-utilization accounts. Our results also indicate that unemployment and economic downturns play a quantitatively small, or even irrelevant, role in the changes in account balance associated with changes in an account's delinquency status and in the account balance's EAD. The impact of a downturn on the recovery rate and LGD is found to be large. These findings are particularly relevant for the analysis of bank regulatory capital required under the IRB approach proposed in the Basel II accord.

JEL classification: G20; G32; G33
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1. Introduction

A number of studies have contributed to our understanding of credit risk in credit card portfolios; however these studies were not intended to provide a comprehensive analysis of portfolio credit risk. This study conducts such analysis and is the first one to employ contemporaneous data from the most severe downturn ever experienced in this area of consumer finance. We analyze all components of credit risk: the probability of default, the exposure at default and the loss given default. Furthermore, while most studies focus on the event of transition to default, we analyze transitions across several delinquency states. We also analyze changes in account balances across a rich set of delinquency transitions—an important component of credit risk and of eventual credit loss from defaulted accounts. Significantly, unlike prior studies, ours analyzes the heterogeneous risk impact of macroeconomic factors across credit card accounts with different risk profiles, which is particularly important for industry practitioners.

Early empirical studies of credit card defaults did not consistently find a significant impact from macroeconomic factors on the probability of transition to default. In more recent studies, the unemployment level is shown to be a significant determinant of default, but the change in unemployment is usually found to be insignificant. Our results provide new evidence of unemployment's impact on credit risk and its different components. In particular, taking advantage of significant variation in policy variables, risk exposure, and performance outcomes experienced during the Great Recession, our analysis indicates that both the level and change in unemployment have a statistically and materially significant positive impact on the likelihood of transition to a higher delinquency state across all transitions considered: current and low utilization, current and high utilization, delinquency, and default. In addition, we find the impact of unemployment across delinquency transitions to be heterogeneous and proportionally larger for accounts with higher credit scores, which represent the bulk of accounts in most banks' credit cards portfolios.

Macroeconomic downturns also have a significant negative effect on the lender's recovery rate from defaulted accounts and, by symmetry, on the loss given default. In contrast, our results indicate that unemployment plays a quantitatively small or even irrelevant role in the changes in account balance associated with an account's delinquency transitions. Other results, including the impact of account age on the probability of default and the significant impact of other account characteristics such as credit score or account utilization, are consistent with the existing literature. In conclusion, we find that the impact of unemployment on credit risk in credit card portfolios is channeled primarily through its impact on the process of account default and recovery.

Most studies of the impact of the Great Recession on the performance of financial assets have analyzed assets with significant exposures to the housing market, mortgage portfolios in particular. The impact of the Great Recession on non-mortgage retail portfolios, however, should not be ignored. Credit card portfolios in particular represent a substantial portion of the balance sheets of the largest U.S. banks and represent an even larger component of their expected profits, as well as projected losses under conditions of economic stress. Specifically, for credit card portfolios, the net charge-off rate increased more than twofold for a number of banks during the peak of the financial crisis, and more than $160 billion is estimated to have been charged off since 2008.[1]

We analyze a panel data set containing account-level credit card information from a random sample of individuals with a credit file in the Equifax credit bureau database. As in previous studies, the probability of default in our model is a function of origination cohort, account age, economic variables, and other control variables that measure the account's inherent risk. Rather than focusing only on the default outcome, we propose a multiple-state model that considers the current, delinquent, and default states and separately considers current accounts with medium-low or high utilization rates. The econometric framework considered is a multi-period-multinomial logit specification. Also, we estimate separate models for current and delinquent accounts at the time of observation because these two groups are expected to perform very differently. Further, we analyze the balance exposure at default for accounts that transition into default, measuring the change in account balance between the time of observation and the time of default. Specifically, linear statistical models are employed to determine the balance exposure of accounts at the time of default, a certain point into the future, expressed as a percentage of the account balance at the time of observation and conditional on the account's characteristics and delinquency status. Given the lack of useful account-level information on recoveries in the credit bureau data, the analysis of LGD, the third component of net credit loss, is not performed at the account level. We perform a multivariate analysis of LGD using recovery and charge-off data reported to regulators in FR Y-9C filings by a select group of U.S. bank holding companies, which allows us to assess the impact of macro factors on LGD. This select group comprises the nation's largest credit card issuers and manages more than 80% of existing credit cards accounts.

Banks regularly use statistical models of projected credit loss as risk management tools. These models provide key inputs into determining economic and regulatory capital as well as for determining the allowance for loan and lease losses (ALLL). Our findings are particularly relevant for the analysis of regulatory capital under Basel II's internal ratings based (IRB) framework, which requires the quantification of credit risk parameters for computing regulatory capital over a mix of economic conditions, including downturn economic conditions.

In the next section we identify a few papers relevant to our analysis. In section 3 we present the data and conduct a descriptive statistical analysis. In section 4 we present the empirical methodology. In sections 5 and 6 we discuss results and offer conclusions, respectively.

2. Literature Review

A number of researchers have analyzed credit cards default. Gross and Souleles (2002), using

[1] As reported in bank call reports, see also Hunt (2013). The net charge-off rate peaked at the time of the financial crisis, propelled by an increase in the net dollar charge-off rate and a decline in dollars outstanding.

panel data on credit card accounts, analyze credit card default and personal bankruptcy, focusing on the 1990s. The empirical model employed in their analysis is of particular interest because it has been adopted by subsequent papers in this literature. The outcome of interest in their study, or the dependent variable, is a dichotomous variable that takes a value of one in a particular month if the credit card account defaults in that month and zero otherwise. An account is considered in default if it is seriously delinquent, which is defined as three monthly billing cycles. Using this variable, they model the delinquency behavior over time of credit card accounts using multi-period probit and logit models, which can also be referred to as discrete time duration models. Their analysis suggests that the probability of delinquency increases from the time an account is booked until about its two-year tenure and then declines. Important predictors of default are low credit score, large balances and purchases, or smaller payments. The authors conclude that the relation between default and economic fundamentals appears to have changed substantially over the period of study in ways not explained by their control variables.

Agarwal and Liu (2003) examine credit card delinquency and bankruptcy behavior using the same econometric framework of Gross and Souleles. They note that previous empirical studies did not consistently identify a significant effect of macroeconomic factors on bankruptcy, while Gross and Souleles did not find a significant impact of unemployment on credit card default. Agarwal and Liu hypothesize that the lack of a significant effect from unemployment may owe to a lack of sufficient variation in the data, due to either inadequate data or insufficient variation in the unemployment variable during the period of analysis. Using data from a large sample of credit card accounts over an extended time frame that includes periods of economic expansion and recession, the authors provide evidence that unemployment has a significant impact on credit card delinquency. Their analysis indicates that the level of unemployment appears to be a significant determinant of default, while the change in unemployment is usually insignificant.

Our empirical framework also considers the analysis of account balance changes associated with changes in delinquency status. It is common industry practice to estimate changes in balances for accounts that transition to default. This type of analysis is also required by Basel II as an intermediate step in the process of computing regulatory capital for credit risk. There is limited relevant literature in this area of inquiry, however. Qi (2009) studied EAD for a sample of current and delinquent accounts over the period 1998–2008 using the incremental accumulated dollar balance of an account at default, usually referred to as the loan equivalent exposure (LEQ).[2] Using this as the analysis variable, Qi found that borrower and account risk attributes such as account utilization rate, account age, account balance, credit score, and credit limit are significant drivers of LEQ. Additionally, Qi found LEQ to be higher in periods when overall default rates are high, which suggests EAD increases in periods when economic conditions worsen. Interestingly, this relationship between worsening economic conditions and EAD was found to be characteristic of the 2002-03 recession. EAD was also found to be the lowest during the 2008 recession, when default rate was the highest, which the author suggests resulted from a reduction in credit card limits by banks. The study makes the important point that the increase in utilization as borrowers approach default can result from either an increase in account balance or a decrease in the line originated by the lender. In our analysis, we avoid this confounding effect by focusing our attention on changes in balance rather than changes in utilization.[3]

Last, we turn our attention to LGD, the third component of net credit loss. Although LGD is an important determinant of credit losses, research on LGD pertinent to retail credit is particularly limited. This is not surprising given the lack of adequate data available to model LGD for retail portfolios. A recent study on LGD was conducted by Bellotti and Crook (2009), who developed a

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[2] The loan equivalent exposure of an account in period *t* that defaults in *t+12* can be defined as the incremental accumulated balance on the account between *t* and *t+12* expressed as a percentage of undrawn balances at *t*.

[3] The exposure at default as a percentage of the outstanding balance at the reference time is usually referred to as the credit conversion factor (CCF).

spate of LGD models using account-level data on major U.K. retail credit cards. They concluded that ordinary least squares models with macroeconomic variables are best for forecasting LGD, at both the account and the portfolio levels. Their findings suggest that higher unemployment is associated with lower recovery rates.

3. Data and Descriptive Analysis

We have access to a panel data set containing trade line credit card information from a 5 percent random sample of individuals with a credit file in the credit bureau database. The data set is maintained by the Retail Risk Analysis unit of the Federal Reserve Bank of Philadelphia. The data set includes up to 10 active credit card accounts per individual. At each observation point, current accounts with zero balance and no activity within the last six months are excluded from the data. For individuals with more than 10 active credit card accounts, a highly infrequent occurrence, the 10 most recently opened accounts are retained.

Table 1
Relevant Variable Definitions

Balance	Current and past-due balance
Line	Credit limit
1500-7500	Dummy, credit line from 1500 to 7500
7500-25000	Dummy, credit line from 7500 to 25000
Utilization	Percentage of the line being utilized
Low U.	Utilization below 35%
Medium U.	Utilization between 35% and 80%
High U.	Utilization above 80%
Payment Status	
Current	Less than 30 days past due
C. zero bal.	Current with zero balance
C. low util.	Current with low utilization
C. medium util.	Current with medium utilization
C. high util.	Current with high utilization
Delinquent	30 days past due up to 89 days past due
Default	90 days or more past due
Credit score	
Score 1	Score up to 560
Score 2	Score 561 to 620
Score 3	Score 621 to 700
Score 4	Score above 700
Re-performing	Current at this time, was delinquent within last 24 months
Fourth Quarter	Dummy for fourth quarter
Account age	Account age from origination date
Acc Age1	Account age is less than one year
Acc Age2 to 5	Account age is 2, 3,…,5 years, respectively
Acc Age6	Account age is 6 years or more
Unemp.	Unemployment rate
Chg. Unemp.	One-year change in unemployment rate

In our analysis, we employ a panel with information on credit card accounts from the end of 2005 to the second quarter of 2010. Thus, our data does not include the period around the passage of the recent bankruptcy reform legislation. For each relevant year, we observe account snapshot information for the months of June and December. Given the enormous size of the original data set, in our analysis we employ a 10 percent random subsample from the sample described above, or a 0.5 percent sample of the overall credit bureau sample.

Table 1 lists the account-level variables available in our sample, as well as any derived variable transformations employed in the empirical analysis. Our data include information on account characteristics such as account age, line, balance, and borrower's credit score, as well as current and past delinquency status. This information is combined with the unemployment rate at the state level, the primary policy variable of interest. We have divided the relevant variables' range of variation into segments as reflected in the table. This segmentation allows us to estimate the potential nonlinear impact of particular variables without having to rely on specific functional-form assumptions.

Table 2
Means and Proportions for Relevant Variables over Time

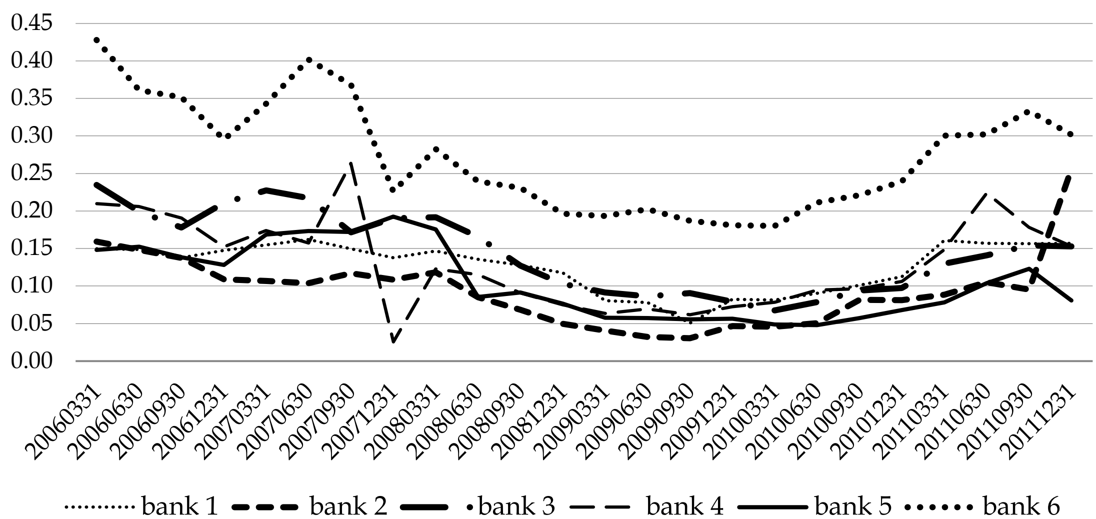
	Y06Q2	Y06Q4	Y07Q2	Y07Q4	Y08Q2	Y08Q4	Y09Q2	Y09Q4
Balance	2102.00	2098.00	2137.00	2168.00	2187.00	2221.00	2186.00	2162.00
Utilization	72.00	71.99	71.81	72.14	72.73	74.65	75.88	76.08
Line	10723.00	10851.00	10961.00	11112.00	11083.00	10983.00	10768.00	10613.00
Line %								
0 – 1500	33.18	34.77	33.97	33.75	33.96	34.58	37.71	38.25
1500 – 7500	36.00	34.91	35.20	34.69	34.77	35.05	35.40	35.91
7500 - 25000	30.83	30.35	30.83	31.56	31.27	30.37	26.89	25.84
Del. %								
Current	96.26	96.43	95.62	95.81	95.44	95.51	95.19	95.95
Low U.	49.10	53.23	50.67	54.13	53.40	50.52	48.18	50.68
Med U.	11.82	12.21	11.42	11.70	10.84	10.77	11.45	12.16
High U.	35.34	30.99	33.53	29.98	31.20	34.22	35.56	33.11
Delinquent	1.31	1.28	1.42	1.33	1.49	1.28	1.41	1.18
Default	2.43	2.30	2.96	2.86	3.06	3.21	3.40	2.87
Score %								
Score 1	9.49	10.47	10.70	11.78	11.26	11.96	12.33	12.78
Score 2	8.79	9.04	9.00	8.79	8.35	8.29	8.12	8.02
Score 3	20.56	20.68	20.28	19.65	18.95	18.50	18.50	18.62
Score 4	61.16	59.81	60.02	59.78	61.44	61.25	61.05	60.59
Score mean								
C. zero bal.	724.00	726.00	725.00	728.00	729.00	732.00	734.00	737.00
C. low util.	629.00	626.00	623.00	619.00	621.00	623.00	627.00	632.00
C. high util.	629.00	626.00	623.00	619.00	621.00	623.00	627.00	632.00
Delinquent	499.00	499.00	492.00	481.00	482.00	482.00	486.00	490.00
Cur. or del.	744.00	741.00	743.00	745.00	748.00	750.00	750.00	750.00
Re-performing	6.93	6.46	6.88	6.56	6.90	6.48	6.76	6.12
Account age	6.66	6.59	6.55	6.52	6.58	6.69	7.02	7.34

Table 2 provides information on average values and proportions for relevant variables for specific variable ranges. For accounts with positive utilization, we observe that average utilization was around 72 percent in the fourth quarter of 2007 (prior to the recession) and increased continuously to about 76 percent in the fourth quarter of 2009. The table also provides information on the distribution of accounts by credit score bands. About 60 percent of accounts are concentrated in the highest credit score band over time. As a result of account migration due to an increase in delinquencies, we observe an increase in the proportion of accounts in the lowest credit score band around the period of economic stress.

Table 2 also provides information on delinquency status over time. Between 95 percent and 97 percent of accounts remain current over a six-month period. Also, the six-month default rate was higher around the time of the recession, with rates of 3 to 3.4 percent. Re-performing accounts, or accounts that have experienced a delinquency of at least 30 days over the last two years, represent between 6 percent and 7 percent of accounts. Finally, the average account age is between six and seven years. We observe an increase in average account age during the period of economic downturn, the result of a lower rate of new account originations during this period.

Next we discuss the data used for the LGD analysis. A time-series view of bank holding company (BHC) recovery rates is shown in Figure I. [4] Recoveries from defaulted accounts are traditionally low in credit card portfolios, often less than 10 percent of losses and up to 20 percent in some instances. The figure makes it apparent that the recovery rate at banks included in our sample declined over the period of the Great Recession. The overall average recovery rate is 15 percent, while the average recovery rate over the recession quarters (first quarter of 2008 to the first quarter of 2010) is significantly lower at around 10 percent.

Figure I
Recovery Rates over Time at Large Financial Institutions in the U.S.



Source: Bank call-report data.

4. Empirical Methodology

We consider a traditional framework for the credit risk analysis of a credit card portfolio. This framework takes into account three components of risk: the probability of default (PD), the exposure at default (EAD), and the loss given default (LGD), which represents the percentage of the exposure that is lost at default. Expected loss can be defined as the product of these three components:

[4] Source: BHC FR Y-9C submissions. Reported FR Y-9C data on recoveries and charge-offs may include recoveries and charge-offs on small business card portfolios in addition to consumer credit cards.

$$EL = PD \cdot LGD \cdot EAD. \tag{1}$$

It is normal industry practice to consider the analysis of each of these components of loss separately. This practice has also been solidified by the implementation of this traditional framework as part of the process of analyzing credit risk under Basel II. Next, we describe the econometric methodology considered in the analysis of gross credit loss and the PD and EAD parameters in particular. The lack of useful information on recoveries in our credit bureau data set prevents us from also conducting an account-level analysis of LGD, the third component of net credit loss. For the analysis of LGD we resort to using more aggregated, publicly available, information from regulatory filings. The empirical methodology is described in the next subsections.

4.1. The Probability of Default and the Process of Delinquency Transitions

We assume that a credit card account can be in one of several current or delinquent states at each particular point in time. We model delinquency as a process of transition across states over time with default representing an absorbing state. At each point in time, delinquency status is a function of account characteristics, customer characteristics, economic environment, and delinquency history up to the present time. In particular, assume that at time t a credit card account can be in one of K+1 possible delinquency states $s_t \in \{d_0, \dots, d_K\}$. For a particular credit card account n, denote the relevant risk drivers at time t, including delinquency history up to time t, as $R_n(t)$. For accounts active at time t, we assume a suitable multinomial logit probability specification for the transition from the present state at time t to any alternative state six months into the future, at time $t+1$. Transition probabilities are defined as follows:

$$P\left(s_{t+1} = d_k | s_t = d_j, R_t\right) = \frac{exp\left(\varphi_{jk}(R_t)\right)}{1+\sum_{i=1,\dots,K} exp\left(\varphi_{ji}(R_t)\right)}, for\ k = 0, \dots, K, \tag{2}$$

or $\Pr\left(\mathrm{d_k}|\mathrm{d_j}, \mathrm{R_t}\right)$ for simplicity. In particular, we consider the following convenient specification:

$$\begin{gathered}\varphi_{ji}(R_t) = \lambda\left(age(t), \beta_{ji}^{\lambda}\right) + \delta\left(R_t, \beta_{ji}^{\delta}\right), for\ k = 1, \dots K \\ and\ \varphi_{j0}(R_t) = 0,\end{gathered} \tag{3}$$

where $\lambda\left(age(t), \beta_{ji}^{\lambda}\right)$ represents a baseline hazard of account age with a semi-parametric specification (in the spirit of Han and Hausman, 1990; Meyer, 1990; McCall, 1996; and Deng, Quigley, and Van Order, 2000). The factor $\delta\left(R(t), \beta_{ji}^{\delta}\right)$ captures the effect of risk drivers and the account's delinquency history, and, in our empirical framework, $\delta\left(R(t), \beta_{ji}^{\delta}\right)$ takes a linear specification form for simplicity and convenience of interpretation. The coefficients $\left(\beta_{ji}^{\lambda}, \beta_{ji}^{\delta}\right)$ are specific to the origination and destination delinquency states. The condition $\varphi_{j0}(R_t) = 0$ is consistent with the standard multinomial logit specification (Green, 2002). Within this framework, the contribution to the sample likelihood of account n with account history $\{(d_{nt}, R_{nt}), t = 1, \dots, T\}$ is

$$\prod_{t=1,\dots,T} P(d_{nt+1}|d_{nt}, R_{nt}). \tag{4}$$

We obtain the following expression for the likelihood function for a sample of N accounts:

$$\prod_{n=1,\dots,N} \prod_{t=1,\dots,T} P(d_{nt+1}|d_{nt}, R_{nt}). \tag{5}$$

Rearranging terms, we obtain an equivalent expression of the form

$$\left\{\prod_{t=1,\dots,T}\prod_{n_0=1,\dots,N_{0t}} P\left(d_{n_0t+1}|s_t=0,R_{n_0t}\right)\right\}\cdot\ldots$$
$$\cdot\left\{\prod_{t=1,\dots,T}\prod_{n_K=1,\dots,N_{Kt}} P\left(d_{n_Kt+1}|s_t=K,R_{n_Kt}\right)\right\} \quad (6)$$

for each $k=0,\dots,K$, where each component describes the likelihood function for the transition from state $s_t = k$ to any other state s_{t+1} within a multinomial logit specification. This expression indicates that, as long as there are no common unobserved elements across the different components or unobserved heterogeneity, the MLE associated with this specification will be equivalent to considering $K+1$ panel multinomial logit specifications, with each of these specifications independent of one another.

Multi-period-multinomial logit specifications can be interpreted as a particular type of discrete time duration model.[5] Shumway (2001) makes this point theoretically. In particular, Proposition 1 in that paper indicates that "a multi-period logit model is equivalent to a discrete-time hazard model [under certain distributional assumptions]." Sueyoshi (1995) makes a similar point.

Our model specification incorporates all the basic ingredients employed in the relevant literature discussed in the introduction. The advantages of this specification are the ease of interpretation, as illustrated elsewhere in this paper, and its ideal numerical properties.[6] Several econometric studies indicate that the use of a flexible specification to account for time dependency, and time dummies in particular, goes a long way toward minimizing the impact of spurious unobserved heterogeneity, which is necessarily present in any econometric model. This is also the approach we employ.[7]

4.2. Exposure at Default and the Balance Ratio

In the four-state transition model used for our analyses, each non-defaulted account at t (current with low utilization, current with high utilization, and delinquent) can transition into one of the four possible delinquency states in $t+1$, six months into the future, with default included as the terminal state. As a result, there are 12 transition states, and the projected exposure of accounts corresponding to each transition has to be determined.

Typically, econometric models used to estimate the amount of exposure of defaulting accounts are referred to as models of exposure at default (EAD); these models have been the primary focus of the academic literature. The transitions from current to default and from delinquent to default are important from the perspective of credit risk management and loss projection. On the one hand, current accounts have a low risk of default, but they traditionally constitute the lion's share of a credit card portfolio and can contribute the largest balance increases at default. On the other hand, delinquent accounts in a well-managed portfolio are likely to contribute only relatively modest future balance increases, but they typically have a high probability of default.

We model balance changes for account transitions using a "balance ratio," or BR, approach. The balance ratio for a particular account at time t is defined as the ratio of the account balance in period $t+1$ to the account balance in period t. The econometric approach to estimating changes in the BR considers a log-linear model specification, with the log of the balance ratio as the dependent variable,

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[5] Literature surveys of duration models include Kiefer (1988), Canals-Cerdá and Stern (2002), and Van Den Berg (2009).

[6] More precisely, models in the logit family have the property of global concavity of the likelihood function, which guarantees convergence of the maximum likelihood estimator to the optimum (Amemiya, 1985).

[7] Early proponents of this approach include Han and Hausman (1990), Meyer (1990), and McCall (1996). In particular, the model developed in McCall (1996) to analyze unemployment was subsequently applied in an influential study of mortgage prepayment and default by Deng, Quigley, and Van Order (2000). The presence of unobserved heterogeneity would bring to bear additional computational challenges (Heckman and Singer, 1994; Baker and Melino, 2000; and Canals-Cerdá and Gurmu, 2007) and conceptual challenges (Heckman, 1981; Cameron and Heckman, 2001; and Bearse, Canals-Cerdá, and Rilstone, 2007).

$$Log(BR)_{jit} = \varphi_{ji}(\mathrm{R}_{\mathrm{jit}}, \mathrm{t}) + \varepsilon_{jit}, \quad (7)$$

where i and j represent the account's state at period t and $t+1$, respectively, and $\varphi_{ji}(\mathrm{R}_{\mathrm{jit}}, \mathrm{t}) = \alpha_{ji} + \beta_{ji}R_{jit}$ represents a general linear specification considered in our empirical framework, with R_{jit} representing the independent variables, or risk drivers, from the set of potential variables defined in Table 1. The model also includes interactions across risk drivers for some empirical specifications, ε_{jit} represents other account- and time-specific idiosyncratic factors.

4.3. Loss Given Default

Credit bureau data do not include detailed account-level information on recovery rates from defaulted loans. For this reason, we performed an analysis of LGD, the third component of net credit loss, using recovery and charge-off data reported by a select group of U.S. BHCs in their FR Y-9C regulatory reports. Overall the BHCs included in the analysis accounted for over 80 percent of U.S. credit card receivables at the end of the third quarter of 2010.

We model the recovery rate as a simple autoregressive process. Suppose $\mathrm{RR}_{\mathrm{it}}$ is the recovery rate of the *i-th* bank at time t. Then $\mathrm{RR}_{\mathrm{it}}$ is modeled as

$$RR_{it} = \mu_i + \beta RR_{it-1} + \gamma M_t + \varepsilon_{it}, \quad (8)$$

where μ_i is the mean recovery rate of the i-th BHC and ε_{it} is the white-noise error term. Our model implies that banks' recovery rate is a stationary stochastic process that is mean reverting, and deviations of recovery rate from the mean in any given period are explained by the recovery rate in the recent past and variables exogenous to the process—here, a variable that captures the macroeconomic trend. Note that LGD can be determined once the recovery rate is estimated using the relationship $LGD_{it} = 1 - RR_{it}$. During practical implementation, application of this LGD factor, at the loan level, to gross portfolio losses would yield net portfolio losses.

5. Analysis of Empirical Results

We apply the theoretical econometric framework described in the previous section to analyze credit risk in a generic credit card portfolio. Estimation results for models of delinquency transition, results for balance ratio models of exposure at default, and results for the recovery rate, or equivalently LGD, are presented in the following subsections.

5.1. Probability of Default and the Process of Delinquency Transitions

Each model specification analyzed considers the probability of transition from the present state at observation time to any one of several possible states six months into the future. The most simple model specifies a framework in which accounts transition across three possible delinquency states (current, delinquent, and default). We also consider an extension of this model in which the current state is further segmented into two distinct states according to the account's line utilization level: current accounts with high utilization and other current accounts not in that group. We present results for this second, more comprehensive model specification because it adds to our understanding of the impact of macro variables on the delinquency transition process and contributes some additional insights to the analysis of credit risk.[8]

We experimented with a variety of model specifications before selecting the final ones reported. It is worth noting that we observed a high correlation between delinquency projections across different sensible model specifications. Model risk drivers include line utilization, re-performing status, a fourth-quarter dummy, vintage, account age, credit score, and two policy variables representing the unemployment rate and the change in unemployment rate. The unemployment variables are lagged three and six months, respectively, with respect to the time at which the delinquency outcome is reported. There is a high correlation between both measures of unemployment; one should consider this fact when interpreting the associated model parameters.

Estimation results for models with state fixed effects are presented in Table 3 and are discussed

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[8] Readers interested in the results from the simpler model specification can request this information from the authors.

next. Models without fixed effects offer similar results and are available from the authors. Parameter estimates of nonlinear models are inherently difficult to interpret; to facilitate this task, parameter estimates are reported as odds ratios.[9] A convenient feature of the multinomial logit model is that the odds ratio coefficients are invariant across values of the explanatory variables, much like the coefficients in a linear regression model. When represented in the form of odds ratios, a parameter estimate above or below 1, respectively, represents an increase or a decrease in the odds of a particular outcome as a result of an increase in the value of the associated explanatory variable. Given the huge size of the sample employed in our paper, most parameters with associated odds ratios that deviate even slightly from 1 will be statistically significant.

We present results for a model specification that includes interactions between unemployment and credit score. The inclusion of vintage effects does not have a significant impact on other parameters. Simpler model specifications that do not include vintage dummies or interactions between unemployment and score produce similar qualitative results and are available from the authors. The first three columns in Table 3 present parameter estimates for the transition from the current and low-medium utilization state at time t to a higher utilization/delinquency state at time t+1, six months into the future. We observe that the highest credit line group is associated with an increase in the odds of transition to the default state and a decrease in the odds of transition to other states, compared with the odds of remaining in the current and low-medium utilization state. A medium utilization rate is associated with higher odds of transition to the delinquent state, and lower odds of transition to the default state, than a high utilization rate. Not surprisingly, re-performing accounts are at a high risk of transition to the delinquent and default states, as indicated by the associated odds ratios of 3.6 and 2.2, respectively. Vintage effects are not reported in the table, but we would say that the 2007 vintage in particular is associated with an increase in the odds of transition to the delinquent and default states.

Account age enters the model in the form of age dummies, allowing for a great deal of flexibility on its effect. Taking as the control group accounts with an age of less than one year, we observe that the odds of transition to a current and high-utilization state, as well as the odds of transition to the delinquent or default states, decrease with account age. This result indicates that new accounts are more risky than more seasoned accounts, after controlling for other drivers of risk, and is broadly consistent with previous studies (e.g., Gross and Souleles, 2002). This result is particularly relevant for assessing the latent risk of account origination strategies by financial institutions.

Not surprisingly, credit score is an important determinant of the probability of transition to the delinquent and default states. For a current account in the second-lowest credit score group, the odds ratios for the transition to the delinquent or default states are 0.55 and 0.21, respectively. These values imply a twofold increase in the odds of delinquency and a fivefold increase in the odds of default for accounts in the lowest credit score segment compared with accounts in the second-lowest credit score segment. As expected, the results are even more pronounced when we compare accounts in the lowest credit score segment with accounts in the two highest credit score segments. In particular, for the second-highest credit score segment, the odds ratios are 0.20 and 0.03, and for

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[9] Observe that the odds of a particular outcome k relative to the base outcome are defined by the expression

$$\frac{exp(\beta_k x)}{1+\sum_{i=1,\dots,K} exp(\beta_i x)} : \frac{1}{1+\sum_{i=1,\dots,K} exp(\beta_i x)} = exp(\beta_k x)$$

and can be denoted as $Odds_k(x)$. We can consider the odds ratio of increasing a certain characteristic x_i by a unit as equal to

$$\frac{Odds_k(x_{-i}, x_i+1)}{Odds_k(x)} = \frac{exp(\propto + \beta_{-i}x_{-i} + \beta_i x_i + \beta_i)}{exp(\propto + \beta x)} = \exp(\beta_i),$$

where $x = (x_{-i}, x_i)$, $\beta x = \beta_{-i}x_{-i} + \beta_i x_i$, and the odds ratio is independent of characteristics and equal to $exp(\beta_i)$. The odds ratio approach accepts a simple interpretation: If a certain variable has no material impact on the odds of a certain outcome, we would expect the associated odds ratio to be about 1. Reported t-values are relevant for the null hypothesis of an odds ratio equal to 1. If a certain variable has a positive impact, the odds ratio will increase above 1, and if it has a negative impact, the odds ratio will decrease below 1. For example, if a one-unit increase in a particular variable doubles the odds of a sale, then the odds ratio will be equal to 2.

the highest credit score segment, the odds ratios are 0.04 and 0.002, respectively.

Table 3
Parameter Estimates for the Four-State Transition Model with Fixed Effects

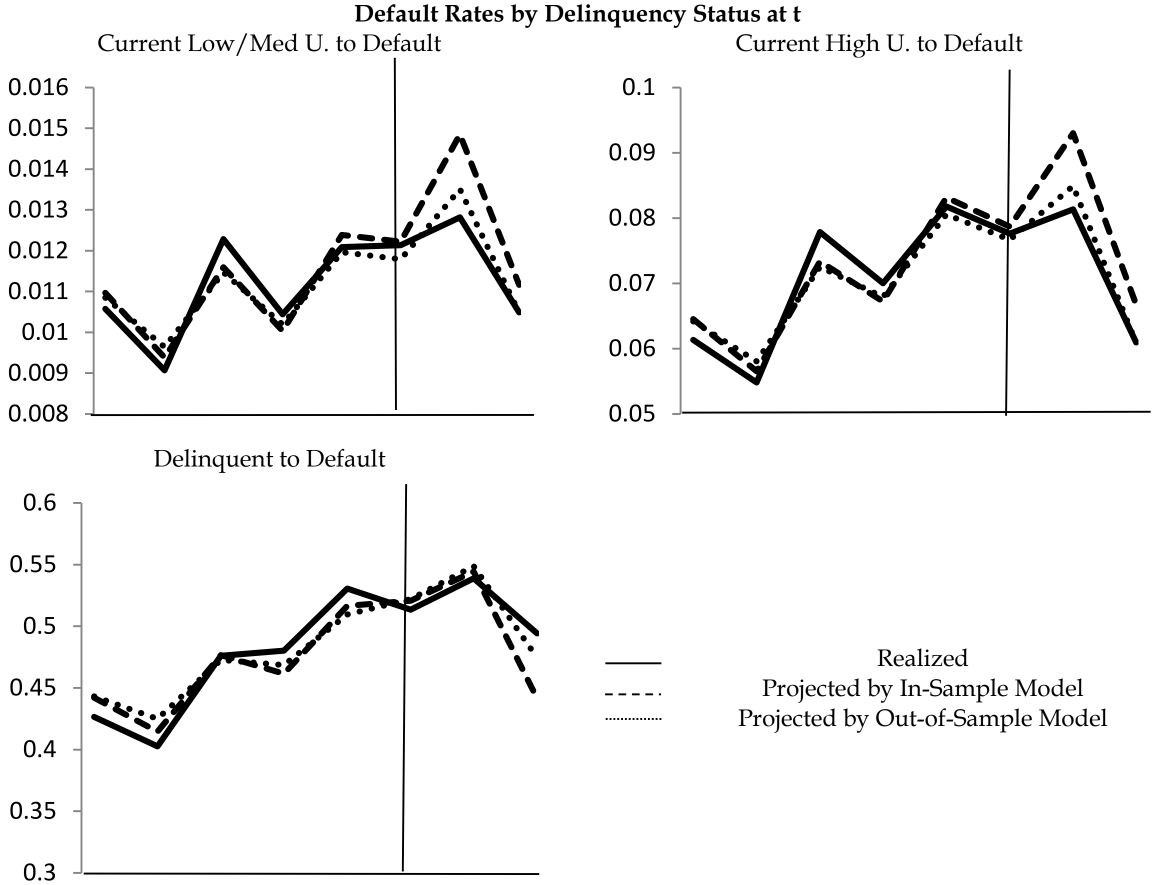
	Current & Low/Med U.			Current & High U.			Delinquent		
	Curr.+	Del.	Def.	Curr.+	Del.	Def.	Curr.+	Del.	Def.
1500-7500	0.77***	0.97***	0.97***	1.39***	1.43***	1.51***	1.41***	1.46***	1.36***
7500-25000	0.58***	0.94***	1.10***	1.33***	1.86***	2.43***	1.46***	1.59***	1.74***
Medium U.	0.74***	2.33***	0.68***				0.63***	1.43***	1.58***
High U.							3.98***	3.60***	7.07***
Re-performing	1.32***	3.62***	2.24***	0.66***	1.73***	1.22***			
Fourth Quarter	0.86***	0.83***	0.79***	0.85***	0.73***	0.72***	0.87***	0.70***	0.75***
Acc. Age2	0.81***	0.71***	0.82***	1.26***	0.99*	0.86***	1.02	1.08***	0.66***
Acc. Age3	0.71***	0.64***	0.78***	1.33***	0.92***	0.80***	1.03	1.14***	0.57***
Acc. Age4	0.72***	0.64***	0.79***	1.41***	0.93***	0.82***	1.07***	1.12***	0.56***
Acc. Age5	0.66***	0.62***	0.75***	1.48***	0.94***	0.83***	1.09***	1.20***	0.53***
Acc. Age6	0.60***	0.63***	0.67***	1.69***	0.98**	0.82***	1.07***	1.35***	0.44***
Credit score									
Score2	1.15***	0.55***	0.21***	0.72***	0.36***	0.17***	0.93***	0.69***	0.58***
Score3	0.92***	0.20***	0.03***	0.43***	0.11***	0.03***	0.75***	0.43***	0.31***
Score4	0.65***	0.04***	0.00***	0.28***	0.08***	0.00***	0.61***	0.24***	0.14***
Policy Variables									
Chg. Unemp.	1.01**	1.03***	1.03***	1.04***	1.07***	1.09***	1.03***	0.99	1.07***
Chg. Un. x Score2	1.02***	1.01	1.02***	1.02***	1.03***	1.04***	1.01	1.00	1.01
Chg. Un. x Score3	1.00	1.01	1.05***	1.04***	1.08***	1.09***	1.01	0.99	1.02**
Chg. Un. x Score4	1.01***	0.98***	1.04***	1.04***	1.10***	1.10***	0.97***	0.90***	1.01
Unemp.	0.98***	0.96***	1.00	0.99***	0.97***	0.98***	1.01	0.99	1.03***
Un. x Score2	1.01***	1.01**	1.02***	1.01***	1.02***	1.04***	1.01	1.00	1.02***
Un. x Score3	1.03***	1.04***	1.05***	1.04***	1.05***	1.09***	1.03***	1.01	1.06***
Un. x Score4	1.04***	1.06***	1.08***	1.05***	1.11***	1.18***	1.05***	1.03***	1.10***
LLF		-116328			-3071054			-419894	

Notes: Geographic states and vintage years are also included in the model specification as fixed effects but are not reported. Curr.+ refers to current accounts that have high utilization.

Other important variables to consider are lag-unemployment and lag-unemployment-change, which are interacted with credit score. The results indicate that both measures of unemployment have a positive association with the likelihood of transition to the delinquent and defaulted states. Interestingly, the interaction between credit score and unemployment indicates that different credit score groups respond differently to an increase in unemployment. Because of the high correlation between the unemployment and change-in-unemployment variables, it is not very useful to interpret the parameters associated with these variables separately. In the more complex model, the impact of unemployment, for example, is captured by a baseline parameter that affects all credit score groups and an interaction parameter that represents the incremental impact of unemployment in the specific group with respect to the base credit score group—with the baseline group defined as the group with the lowest credit score. The overall conclusion is that an increase in unemployment has a proportionally smaller impact on the lower-credit-score groups. In particular, lower-credit-score groups have a much higher propensity to default under any kind of economic conditions, but, in relative terms, unemployment has a smaller impact on lower-credit- score groups, as indicated by a smaller change in the odds ratio as a result of an increase in unemployment. Thus researchers and industry practitioners interested in projecting default rates resulting from changes in macroeconomic conditions are advised to consider a modeling framework that can account for the heterogeneous impact of macroeconomic conditions across credit-score ranges.

Columns four to six present parameter estimates for the population of accounts current with high utilization at time *t*. We observe significant differences in parameter estimates compared with those in the first three columns. In particular, both the highest and the second-highest credit line groups are associated with a significant increase in the odds of remaining in the current-high-utilization state and an even larger increase in the odds of transition to the delinquent and default states. Specifically, for high-line accounts, the odds ratios associated with accounts with the highest lines are 1.86 and 2.43 for the transition to the delinquent and default states, respectively. We also observe significant differences in the impact of unemployment on the transition across states. Specifically, we find that higher unemployment has a positive impact on the odds of remaining in the current-high-utilization state and an even larger impact on the odds of transition to the delinquent and default states. Columns seven to nine present parameter estimates for the population of accounts delinquent at time *t*. The results are broadly consistent with our expectations. Accounts in the highest line range are more likely to transition to default and high utilization is an important predictor of transition to default. Another important variable to consider is account age. The odds of remaining in the delinquency state seem to increase with account age, while the odds of transition to default decrease significantly with account age.

Figure II
Default Rates by Delinquency Status at t



Out of sample projections are defined as one-year out of sample to the right of the vertical line in each graph.

As expected, accounts with low credit scores are significantly more likely to transition to the default state and less likely to remain in the delinquent state. We should point out that, for accounts delinquent at *t*, the score variable considered has a six-month lag. Results indicate that the lagged score variable is less likely to be affected by the current delinquency and is more informative. The results are similar when we consider a one-year lag score variable instead.

Both measures of unemployment have a positive association with the likelihood of transition to default, while an increase in unemployment reduces the odds of remaining in the delinquent state. The interaction between credit score and unemployment indicates that, in relative terms, lower-credit-score groups have a lower propensity to transition to the default states as a result of an increase in unemployment, as indicated by a smaller change in the odds ratio.

Figure II compares in-sample and out-of-sample model projections with realized default rates. Both types of projections are depicted over the period 2006 to 2010 and are presented for the three delinquency transitions considered. Out-of-sample model projections are obtained by re-estimating the model with a sample that excludes the last one year of data. Overall, the results do not seem to exhibit a significant or systematic bias. The average bias in the in-sample models is around 3 percent, with the highest bias around 5 percent across models. The average bias for the out-of-sample models is between 3 percent and 5 percent, with the highest bias between 9 percent and 15 percent.

The out-of-sample models seem to exhibit a downward bias across model specifications for the last period considered. Because the economic downturn period in our data is concentrated in the last two years, it should not be surprising that excluding one of these years from the estimation will have an impact on the overall results. We view this as a cautionary tale for risk management, since it suggests that model loss projections may not be conservative enough if the data do not include a sufficiently representative stress period.

5.2. Exposure at Default and the Balance Ratio

Because of the unsecured and revolving nature of credit card lending, the analysis of credit risk in credit card portfolios should also take into account the potential impact of changes in account balances to assess portfolio risk and potential losses. For the purpose of calculating gross expected credit losses, one should determine the risk of account default and the expected dollar amount outstanding of accounts likely to default. An account's dollars outstanding at the time of default is referred to as exposure at default (EAD). The challenge in estimating EAD pertains to the determination of the incremental additional draws on credit lines of accounts that are current or delinquent as of the observation date and that default in the future. We consider as the analysis variable for EAD the balance ratio or dollar balance of accounts at default expressed as a percentage of balance at the observation date. Specifically, as described in section 4.2, BR models are used to determine the dollar exposure of accounts six months into the future expressed as a percentage of the account balance at observation time across the different transition states.

Table 4
Balance Ratio Descriptive Statistics

			Centiles				
	Mean	Std.	20	40	50	60	80
Curr. low/med. u. to delinquent	2.19	2.81	0.97	1.14	1.29	1.49	2.33
Curr. low/med. u. to default	1.82	2.06	1.16	1.16	1.16	1.30	2.01
Curr. high u. to delinquent	1.19	0.43	1.01	1.08	1.12	1.17	1.35
Curr. high u. to default	1.31	0.47	1.14	1.16	1.21	1.28	1.53
Curr. all to delinquent	1.58	1.85	1.00	1.09	1.14	1.23	1.57
Curr. all to default	1.47	1.25	1.15	1.16	1.20	1.28	1.61
Del. to default	1.21	0.34	1.10	1.16	1.16	1.16	1.31

In the four-state transition model discussed previously, each non-defaulted account at observation period *t* (current with low utilization, current with high utilization, and delinquent) transitions into one of the four possible delinquency states at *t+1* (current with low utilization, current with high utilization, delinquent, and default). As a result, we consider 12 transition states and determine the exposure of accounts for each transition. From a modeling perspective, we estimate dollar exposures of current and delinquent accounts separately for the defined terminal states. In order to keep the presentation of results within a manageable limit, our analysis of parameter estimates focuses mostly on the discussion of results where default is the terminal state.

Table 4 provides descriptive information on average values and distribution of our analysis variable, BR. We observe that for the delinquent-to-default population, on average, balances increase by about 21 percent between delinquency and default. For current accounts, the average percentage increase in account balances on the path to delinquency or default is much higher. It is about 58 percent for accounts that turn delinquent and about 47 percent for accounts that default. Intuitively, since a typical delinquent account is expected to have limited access to credit as lenders make it more difficult for borrowers to draw funds, the changes in account balances resulting from the delinquent-to-default transitions are expected to be moderate. A further breakdown of current accounts by the level of utilization shows that it is the low-to-medium-utilization accounts that contribute largely to the balance increases at delinquency or default. For low-to-medium-utilization current accounts, balances increase about twofold on the path to delinquency or default. On the other hand, balance at the time of delinquency (default) is higher by about 19 percent (31 percent) for the current high-utilization accounts.

Estimates for the BR models for current and delinquent accounts that transition to default are presented in Table 5. Regression results for current accounts that transition into delinquency are also reported. We do this to lend some insight into the evolution of balances for current accounts that might ultimately default. Specifically, parameter estimates from the regression of current accounts that transition to delinquency and default are presented in the table's first two columns, numbered (1) and (2), respectively. Also, regression results for current accounts that have high utilization (utilization > 80 percent) and that default are shown in column (3), and regression results for the delinquent accounts that default are shown in column (4). In order to ascertain the sensitivity of the BR to the timing of balance builds, we also estimated balance changes of accounts over a one-year period and observe very similar results (available from the authors).

We discuss first the results of the *t*-th period delinquent accounts that default in period *t+1*, six months into the future. Our results suggest that several account characteristics (credit line, utilization, vintage, and account age) and borrower characteristics (credit score) are important determinants of BR and ultimately EAD.

Economic factors do not seem to play a pivotal, or even relevant, role in determining dollar exposure of defaulting accounts. The potential to draw down on undrawn credit lines and increase balances is expected to be greater for accounts that have high credit lines and that have used a relatively low percentage of these lines prior to default. We observe that BR is highest for accounts that have not used much of their credit line, i.e., accounts with utilization less than 35 percent. Relative to these low-utilization accounts, the BR of accounts with medium utilization (35 to 80 percent) and high utilization (above 80 percent) are 8 and 10.6 percent lower, respectively.

Lagged credit score is seen to play a relatively small role in determining BR and, ultimately, EAD. Accounts with a longer tenure have a lower BR. Also, we observe that BR decreases monotonically with credit line: Accounts with credit lines less than $1,500 have the highest BR, while by comparison the BR of accounts with credit lines between $1,500 and $7,500 is 9.7 percent lower and the BR for accounts with credit lines over $7,500 is 13.7 percent lower. These results suggest that two delinquent accounts with the same utilization but different credit lines will behave differently as regards balance ramp-up as both approach default; the low-credit-line account will draw a greater percentage of the undrawn line at default. Overall, we find that credit line and utilization are the main drivers of the BR of delinquent accounts that transition to default. Results for the current-to-delinquent and the current-to-default population shown in columns (1) to (3) are directionally similar to the results discussed above. We observe that the BR is lower for older accounts and accounts that are highly utilized, that have high credit lines, and that have been originated in recent years.

Table 5
Parameter Estimates for EAD Models over a Six-month Horizon

	Curr. to Del.	Curr. to Def.	Curr. + to Def.	Del. to Def.
Models:	(1)	(2)	(3)	(4)
	Coef.	Coef.	Coef.	Coef.
1500-7500	-0.189***	-0.186***	-0.175***	-0.097***
7500-25000	-0.221***	-0.228***	-0.206***	-0.137***
Medium U.	-0.481***	-0.209***		-0.080***
High U.	-0.581***	-0.292***		-0.106***
Re-performing	-0.100***	-0.001	-0.013***	
Fourth quarter	-0.037***	-0.010***	-0.009***	-0.008***
Acc. Age2	-0.102***	-0.085***	-0.06***	-0.010***
Acc. Age3	-0.128***	-0.108***	-0.079***	-0.021***
Acc. Age4	-0.154***	-0.119***	-0.088***	-0.024***
Acc. Age5	-0.146***	-0.122***	-0.088***	-0.029***
Acc. Age6	-0.183***	-0.142***	-0.094***	-0.034***
L. Score2	-0.025***	0.019***	0.005***	0.013***
L. Score3	-0.075***	0.007***	-0.014***	0.016***
L. Score4	-0.169***	-0.020***	-0.035***	0.005*
Chg. Unemp.	0.008***	0.005***	0.005***	0.001
Unemp.	-0.003***	-0.002***	-0.002***	0.001**
Intercept	1.006***	0.729***	0.418***	0.326***
R-sq.	0.116	0.143	0.173	0.056

Notes: Geographic states and vintage years are also included in the model specification as fixed effects, but are not reported.

When compared to delinquent accounts, the marginal impact of utilization, credit line, and age on the BR is much larger for current accounts. The marginal impacts of these variables on the BR are also larger for the current-to-delinquent population than for the current-to-default population.

As expected, the results suggest that for accounts that default the delinquency state at the observation date and the account's terminal state are both important determinants of the BR. Since accounts in later stages of delinquency are subject to rigorous monitoring by banks that typically make it less easy for such accounts to draw funds, it is expected that current defaulting accounts will have a higher BR compared to defaulting delinquent accounts but a lower BR compared to current accounts transiting into delinquency.

Again, not surprisingly, current accounts that have experienced delinquency at least once in the past 24 months have a lower BR. For the current-to-delinquent population, parameter estimates on lagged credit scores suggest that better-quality borrowers have lower BRs, and for the defaulting population borrowers with higher credit scores have a lower BR.

Finally, we focus our attention on the impact of the unemployment rate. It is expected that in bad economic times, particularly in periods of high unemployment, borrowers under financial stress would likely increase their use of unsecured, unused, credit lines adding to EAD. Interestingly, we observe that while the impact of both the unemployment level and the change in unemployment on BR is positive and statistically significant, the magnitude of the impact on the BR, and hence on the EAD, is very small with an order of magnitude of less than 1% increase for a 1% increase in unemployment rate or unemployment rate change. These findings suggest that economic conditions do not play a relevant role in determining the dollar exposure of defaulting accounts. We considered a variety of alternative model specifications that rendered the same basic conclusions (not reported here).

5.3. Recovery and Loss Given Default

Under the Basel II capital accord, banks must estimate expected loss given default under stressed economic conditions. In order to quantify the magnitude of the impact of macro factors on bank recovery rates, we estimated the recovery rate (RR) model discussed in Section 3.3 based on publicly available data from regulatory reports and using the unemployment rate and the change in the unemployment rate as the macroeconomic variables of interest, both lagged one quarter. The regression results corresponding to four different RR model specifications are shown in Table 6.

Recovery rates are expected to be lower when the economy is under stress because economic downturns, which are typically associated with higher unemployment rates, reduce borrowers' ability to repay debt. Our findings are broadly consistent with this view. Our results also suggest that a rapid increase in unemployment makes it more difficult for banks to recover losses.

Table 6
Parameter Estimates for Recovery Rate Model

Models:	(1)	(2)	(3)	(4)
	Coef.	Coef.	Coef.	Coef.
Intercept	0.041***	0.072***	0.066***	0.123***
1 qtr lag recovery rate	0.725***	0.647***	0.600***	0.445***
1 qtr lag unemp. rate		-0.003		-0.005***
1 qtr lag chg in unemp. rate			-0.036***	-0.041***
R-sq.	0.770	0.770	0.800	0.810
Num. of observations		144		

Notes: Firm-specific effects are also included in the model specification as fixed effects, but are not reported

6. Concluding Remarks

Using a credit bureau's panel data on credit card account characteristics and performance, we develop an empirical framework for the analysis of credit risk and the projection of credit losses for a generic credit card portfolio. We are not aware of other publicly released studies that undertake a systematic analysis of credit risk for this asset class. We are also not aware of other studies that employ data covering the most severe downturn experienced in this area of consumer finance. Our analysis benefits from a significant variation in policy variables, risk exposure, and performance outcomes, that was not present in data sets analyzed in prior studies. Our work substantiates results from the existing literature and identifies new ones.

Our results indicate that the unemployment rate—in particular, the level and change in unemployment—plays a significant role in the probability of transition across delinquency states in general and the probability of default in particular. The impact of unemployment is heterogeneous across accounts with different credit scores and utilization levels. Our estimates indicate that lower-credit-score groups have a much higher propensity to default; however, in relative terms, unemployment has a smaller impact on these groups of accounts, as indicated by a smaller change in the associated odds ratio of transition to default as a result of an increase in unemployment. Also, the impact of unemployment on the risk of future delinquency and default for accounts current at the time of observation is particularly large for high-utilization accounts. The heterogeneous response to macroeconomic shocks across accounts with different risk profiles is of great importance for industry practitioners, but it has been ignored in the existing literature.

Our findings also indicate that unemployment rate plays a quantitatively small or irrelevant role in the changes in account balance associated with changes in an account's delinquency status. We also considered model specifications that include time-quarter dummies in order to ascertain potential systematic deviations in balance changes at specific calendar dates, after controlling for observable individual risk factors. For these model specifications, our results indicate no relevant differences in account-balance changes by calendar date. Lastly, our analysis of recovery rates indicates that macroeconomic downturns have a negative and significant effect on recovery rates and the associated loss given default.

In conclusion, our results indicate that the impact of unemployment and economic downturns on credit risk in credit card portfolios is channeled primarily through their impact on the process of account default and recovery. Because of the emphasis placed on accounting for economic downturns on the risk parameterization process within the Basel II IRB framework, our findings are particularly relevant for the analysis of credit risk in banks' regulatory capital.

With minimal changes, our framework can be a useful risk management tool for analyzing loss in portfolios other than credit cards. In particular, home equity lines of credit share many similarities with credit cards. Like a credit card, a home equity line represents a line of credit to the customer. Unlike a credit card, in the case of a home equity line the loan is secured by collateral, which may play a significant role in a borrower's likelihood of default and in the potential recovery from a defaulted account.

The analysis of credit loss under conditions of economic stress is particularly relevant at this time, given the recent financial crisis and the worst downturn experienced by the consumer finance industry since its inception.

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