Revisiting the Bid-Ask Spread via Competitive Search

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In this paper, we set up a competitive search model to re-interpret the existence of the market equilibrium bid-ask spread in a stylized security market, in which market dealers are in charge of posting an instantaneous bid price while investors choose whether to sell their share or not at this price. Different from the conventional asymmetric information based explanations, our search based model emphasizes that since the market dealers provide necessary liquidity to the security market via playing such an intermediary role between actual buyers and sellers, the bid-ask spread charged thereafter should largely be justified as the compensation for the market dealers' endeavor in this process. Our model provides a closed-form bid-ask spread formula which has a capacity to reproduce many empirical observations with respect to the effects of the market dealers' maintenance cost, the dividends payoff etc. on the bid-ask spread. Our model further indicates that the absolute bid-ask spread(in dollars) is positively related to the stock price level while the relative bid-ask spread (in percentage) is negatively related to the stock price level, which well solves the puzzle of the impact of stock splits on stock liquidity without the assumption of asymmetric information.

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1. Introduction

As the traditional capital asset pricing model (CAPM) only considers the market systematic risk that is compensated by the equity risk premium (ERP), it cannot explain a variety of pricing puzzles and anomalies associated with the cross-sectional expected stock returns. Many researchers find that the liquidity risk is one of the most important explanatory factors but ignored in CAPM(Pastor (2003)). While we all can feel that financial assets differ in their liquidity, for instance, some stocks are much easier to trade and the others are not if without a longer waiting time or a larger price impact, the exact meaning of liquidity is rather elusive. The origin of the liquidity difference of different assets is not what we are going to investigate here. Insofar as the liquidity of a financial asset is typically measured by its bid-ask spread in literature(Amihud (1986)), rather, in this paper, we will focus on one correlated but more easily tackled question: why does there exist a bid-ask spread for a traded asset and how to quantify it theoretically?

Although there are some papers focusing on the market microstructure trying to interpret why there exists a bid-ask spread in a security market, including one popular model of the bid-ask spread proposed by Kyle(Kyle (1985)), which is mainly based on the asymmetric information, showing that dealers have to widen the bid-ask spread in order to trade against informed investors, our paper here attempts to treat the bid-ask spread from a different viewpoint, i.e. a search-based angle, which doesn't mean that the issue of asymmetric information between dealers and investors is not important. We, however, deliberately avoid the trace of asymmetric information in our model in order to emphasize the more insightful and more fundamental search and matching characteristics of the market equilibrium of the bid-ask spread phenomenon. The center theme delivered by search theory is that it always takes time for one agent to locate another one if the success of a transaction requires the collaboration of both agents. Thus, the difficulty of identifying the counter party of any transaction (either buying or selling a security) in a security market justifies a role of a market intermediary which is introduced to facilitate the easiness of buying and selling through shortening the time wasted in matching and through dampening the price impact of a larger order.

The above represents the concept of "market search friction" that exists in any asset market, but more saliently exists in the over the counter (OTC) market(Vayanos (2008)) and(Lagos (2009)), the government or corporate bond market, the foreign exchange market, the real estate market and the NASDAQ stock market than the NYSE stock market, where the security transaction is more centralized on the trading floor.

In our competitive search model, we assume that there is only one type of asset (or security) to be traded between market dealers and investors. Each market dealer first posts his or her own bid price (will commit to it without collusion with any other market dealers); since there are many market dealers, there will potentially be many sub-markets and each sub-market is distinguished by its unique posted bid price; in doing so, the market dealers anticipate that investors will enter until investors are indifferent across all open sub-markets. The market equilibrium of our competitive search is: there will, however, be only one sub-market left, such that no market dealer has any incentive to open a new sub-market, i.e. to post a bid price different from the remaining one, meanwhile no investors are willing to enter any newly opened sub-market. The key contribution of our paper to the market microstructure literature is that we emphasize the importance of the search and matching cost in financial markets and further show that the "peer pressure" resulted from competition from fellow market dealers can downplay their role as a market intermediary while they are always willing to charge investors with the highest possible bid-ask spread.

Specifically, our model provides a closed-form bid-ask spread formula based on competitive search in a stylized security market, the simulation results of which are well consistent with the actual security market observations, thus proving the validity of our model.

Our model shows that the maintenance cost has two opposite effects on the bid-ask spread. Its direct effect on the bid-ask spread is positive, which is in agreement with the predication of the inventory holding cost theory of the bid-ask spread. Our model also indicates that the indirect effect of the maintenance cost is negative since higher maintenance cost also represents a more competitive structure for market dealers, leading to a less amount of the bid-ask spread. Overall, the positive direct effect of the maintenance cost on the bid-ask spread dominates the negative indirect effect according to our simulation results.

If we treat the dividends payoff as the "negative" maintenance cost, it is natural for our competitive search based model to acquire the positive effect of the dividends on the bid-ask spread. While asymmetric information based models reach the same conclusion, the underlying mechanism is totally different as those asymmetric information based models presume that there exists a positive relation between the level of information asymmetry and the magnitude of the bid-ask spread. Thus the larger amount of the dividends payoff signals the market with the less information asymmetry, causing the bid-ask spread to shrink.

Our model also explicitly studies the influences of the discount rate and the instantaneous market opportunity on the bid-ask spread. Our model indicates that both the discount rate and the instantaneous market opportunity have a positive effect on the bid-ask spread. The above results are reasonable if we consider the discount rate as the opportunity cost of holding stocks without earning the interest of the market dealer's own fund and if we treat the instantaneous market opportunity as the measure of the overall market uncertainty. More importantly, our model studies the impact of a security's price level on its bid-ask spread, which is closely related to a very important and prevalent financial market phenomenon both in the U.S. and worldwide-***stock splits***. People show great interest in the relationship between stock splits and stock liquidity. Contrary to the confusing explanations and observations originated from varieties of asymmetric information based models, our competitive search model clearly shows that the absolute bid-ask spread (in dollars) is positively related to the stock price level, but the relative bid-ask spread (in percentage) is negatively related to the stock price level. Last but not least, better than conventional market microstructure models, our model has the capacity to determine the total number of market dealers at equilibrium since we don't assume that the total number of market dealers is fixed beforehand when modeling the dynamic interactions between market dealers and investors in a security market. This number can be pinned down by the system via a free entry and exit condition for market dealers, which is also the reason why our model is named as the "competitive" search model.

2. Literature review

2.1. Literature on search theory

Our model is based on search theory which is widely used in macroeconomics to explore the matching behavior between workers and firms. The typical framework is in this way: there are two types of agents, workers and firms; they meet with each other depending on the current market tightness; the pure meeting of each other does not necessarily lead to a signed job contract since the worker would expect that he or she might come across a better job offer if he or she just waits a little longer, in the meantime the firm has the same thought to sign a more productive worker as long as it becomes more patient(Diamond (1984)).

With regard to the search mechanism between workers and firms, there are two key modeling issues which can significantly influence the final jointly search results: (1) how workers and firms meet with each other and (2) how they decide the wage rate (i.e. how they split the total profit from a successful hiring). Minor differences in institution arrangements may lead to distinct predictions. Two possible arrangements have been investigated in search theory literature: one is called "*random search*" in which workers and firms randomly meet with each other and the final wage rate for workers is determined via the generalized Nash bargaining scheme once they meet; the other is called "*competitive search*", a.k.a. "*directed search with price posting*" in which workers' job search activities are not random but directed to specific firms either because those firms post and advertise their offered wages or because those firms are natural focal points due to history or established reputation (Moen (1997), Shimer (1996), Rogerson, Shimer and Wright (2005)).

Duffie (2005)and Weill(2008) introduce ***random search*** models to study many important phenomena in finance. We are the first to resort to ***competitive search,*** to investigate the interactions between market dealers and investors in a stylized security market in order to re-interpret the existence of the bid-ask spread in the market equilibrium.

2.2. Literature on the modeling of the bid-ask spread

Demsetz (1968) is a pioneer to study empirically the determinants of the bid-ask spread. Stoll (1978), Huang and Stoll (1997) provide theoretical models for the bid-ask spread and conceptually decompose the market maker cost into three parts: the order processing cost, the inventory holding cost, and the adverse selection cost. The models by Amihud and Mendelson (1980), Ho and Stoll (1993), and O'Hara and Oldfield (1986) focus on the inventory holding cost. Regarding to the modeling of the adverse selection cost, Kyle (1985), Glosten and Milgrom (1985) are pioneers. For instance, Kyle's theoretical model (1985) assumes that there are two types of investors in the market, the random or noise investors and the informed investors. Although Bollena (2004) proposes a simple and parsimonious model to combine the different components of the bid-ask spread, to the best of our knowledge, there is no such a model except ours that puts the important competitive search and matching friction cost into consideration when modeling the bid-ask spread.

The rest of this paper proceeds as follows: Section 3 sets up our competitive search model; Section 4 derives the main results from this benchmark model; Section 5 analyzes the empirical implications of the model; Section 6 calibrates the model and does the simulation, and Section 7 concludes.

3. Competitive search model

3.1. Assumptions

In a simplified world of a stylized security market with only one type of asset or stock to be traded, we have a couple of market dealers (denoted by f) who publicly post an instantaneous bid price (W), at which many potential investors (denoted by w) would choose whether to sell their share or not at this price. We assume that each investor initially has only one share of the stock inhered from endowment; each market dealer can only serve one transaction at a time and each transaction consists of only one share of the stock.

Furthermore, assume that time is continuous and goes from zero to infinity, both agents (f and w) are risk neutral with the (risk-free) discount rate of r. In addition, even though heterogeneity of agents is more realistic, both agents are assumed to be homogenous in this model respectively.

Let the measure of investors be normalized to 1. During each time period, (1-u) of them sell their shares to market dealers and the left of them (u) don't sell their shares to market dealers. Thus, the measure of market dealers who are occupied by stock transactions has to be (1-u). If we assume that the measure of the market dealers who are idle is v, then the total measure of market dealers for this stock will be (1-u)+v=1-u+v. We define the market tightness as $\theta= v/u$, one key system variable which characterizes the tightness of the market condition, namely, the higher the ratio is, the more the number of idle market dealers and the fewer the number of investors who still have shares of the stocks to be sold out, hence the easier for any investor to sell his or her share if he or she is willing to**.** Another note to be emphasized is that since the basic structure of selling and buying is similar in nature, the following model will only be concentrated on one half of the market activities, i.e. the selling part of the market. Namely, we only consider the relation between market dealers and investors in which market dealers post an instantaneous bid price and investors decide whether to sell the share or not. The modeling of the buying part of the market in which market dealers post an instantaneous ask price and investors decide whether to buy the share or not will not become too insurmountable if the selling part of the market is clearly understood. Therefore, from this viewpoint, our model can be classified into a class of partial equilibrium models.

3.2. General picture

In sum, the general picture is: there are two types of agents continuously interacting with one another in the selling part of the market. They need to match with each other to realize their respective optimal profits.

Investors, whose final aim is to sell one share of the stock they obtain from the initial endowment. If an investor holds the share, each period he or she will extract b units of utility (or money) forever. We can think of one share of the stock as one "Lucas tree" which can produce b units of dividends during each time period and b has the same unit as the bid price W. However, with an arrival rate of m (that is assumed to be an increasing and concave function of the market tightness θ), the investor may change his or her mind and sell the share to a market dealer at any bid price W currently posted in the market. To be stressed here, in the strictest term, our investors' search behavior is not random but directed by the bid price W publicly known.

Market dealers play an intermediary role by posting a bid price of W signaling that they are always available to buy investors' shares at W. Market dealers need to compete with each other for supplying the "liquidity service" to investors, which implicitly determines the number of market dealers this security market can finally support at the market equilibrium. During each time period the market dealer incurs a maintenance cost of a when posting a bid price W in the market.

When the transaction is settled, the market dealer pays the bid price W during each time period to the investor in exchange for the share which can then be sold at the price P by the market dealer later. Since we only consider the selling part of the market, the price P that a market dealer can realize in the other side of the market will be treated as a parameter while the posted bid price W is a choice variable. Thus the price difference between P and W, titled by the bid-ask spread, compensates the market dealer for working as a counter party in any stock selling transaction. In the language of economics, (P-W) can also be consider as the normal profit of the market dealer.

With an arrival rate of λ that follows a standard Poisson process, the market dealer who holds one share from the previous transaction cancels this transaction, i.e. stop paying W afterwards. Different from traditional market microstructure models where the overall market sentiment is represented by the arbitrage transaction opportunities that investors are facing, our model stresses the market sentiment embedded in the market dealers' transaction decisions. Through this delicate mechanism design, our model is endowed with the power to mimic the instantaneous arbitrage opportunities for the market dealer due to the fluctuation of the entire market sentiment.

To be clarified here, the price P at which the market dealer can sell out the share, the bid price W posited by the market dealer, the maintenance cost a occurring to the market dealer when posting a bid price in the market, and the dividends payoff b produced by one share of the stock are not "stock variables" but "flow variables" in response to the time factor in our model. Both market dealers and investors are constrained by the basic market structure (the market tightness θ and the functional form of the matching function, m) and the instantaneous market opportunities λ. Whether each selling transaction can be realized mainly depends on whether it is profitable or not for both parties.

3.3. Mathematical model

In this sub-section, we will establish the basic mathematical equations to model the interactions between investors (w) and market dealers (f) in the selling part of the security market.

We first define four key value functions since there are two types of agents (*f* and *w*) and each agent can be in two states (U means that the agent is in the idle state, V means the agent is in the occupied state), the exact meaning of the four value functions are explained below:

$\boldsymbol{U_f}$: the value of a market dealer who posts a bid price and waits for a business;

$\boldsymbol{V_f}$: the value of a market dealer who buys one share from an investor ;

$\boldsymbol{U_w}$: the value of an investor who keeps one share in hand and waits for a chance to sell the share to a market dealer;

$\boldsymbol{V_w}$: the value of an investor who sells one share to a market dealer.

For any posted bid price *W*, the above four value functions satisfy the following four competitive search equations:

$$rU_w = m(\theta)(V_w - U_w) + b \tag{1}$$

$$rV_w = W + \lambda(U_w - V_w) \tag{2}$$

$$rV_f = P - W + \lambda\left(U_f - V_f\right) \tag{3}$$

$$rU_f = [m(\theta)/\theta]\left(V_f - U_f\right) - a \tag{4}$$

Assume the free exit and entry for any market dealer, we have the below free entry condition for the market dealer:

$$U_f = 0 \tag{5}$$

The market equilibrium is characterized by a solution to the following optimal problem:

$$\begin{aligned}(W^*, \theta^*) &= argmaxU_f(W, \theta)\\ \text{s.t. (1) } U_w(W, \theta) &= U_w(W^*, \theta^*)\\ \text{(2) } U_f(W^*, \theta^*) &= 0\end{aligned} \tag{6}$$

4. Discussions

Definition1 (Symmetric Equilibrium): *If (W*, θ*) solve the optimal problem (6), then (W*, θ*) define a market symmetric equilibrium.*

The underlying meaning of the above optimal problem is that at a predetermined market condition ($\boldsymbol{\theta^*}$), given that all other market dealers post *W**, any agents (either market dealers or investors) have no incentive to create a new market with a posted *W* which is different from *W**. This market equilibrium definition is well consistent with the concept of Nash equilibrium and resembles the operation of the actual security market. Moreover, since all market dealers play the same strategy, i.e. post the same bid price *W** throughout the market, we call this type of market equilibrium as the symmetric equilibrium.

Proposition 1: *The market symmetric equilibrium can also be characterized by the below optimal problem of (7), i.e. (6) and (7) are equivalent with each other and will give the same pair of (W*, θ*).*

$$\begin{aligned}(W^*, \theta^*) &= argmaxU_w(W, \theta)\\ \text{s.t. } U_f(W^*, \theta^*) &= 0\end{aligned} \tag{7}$$

Linking Equation (1) to (6) together, we can solve for the four value functions (U_f *, V_f *, U_w *, V_w*) and the two variables (W^*, θ^*) when assuming (r, λ, P, a, b, and the functional form of m) are all exogenous.

According to our equilibrium definition, the two values of (W^*, θ^*) exactly characterize the entire system. They can be pinned down from two reduced equations summarized in Proposition 2.

Proposition 2: *The original six-equation system (Equation (1)-(6)) can be reduced into two fundamental equations: Equation (8) is the free entry equation and Equation (9) is Nash equilibrium equation, which can then be used to solve for (W^*, θ^*).*

Free entry equation:

$$m(\theta^*)(P-W^*)-a(r+\lambda)\theta^*=0 \tag{8}$$

Nash equilibrium equation:

$$m'(\theta^*)(P-b)-a[r+\lambda+m(\theta^*)-\theta^* m'(\theta^*)]=0 \tag{9}$$

Deriving a closed-form formula for the bid-ask spread at the market equilibrium for our stylized security market under the framework of competitive search is one of the most important objectives of our paper. Once (W^*, θ^*) are solved via Equation (8) and (9), the corresponding bid-ask spread in our model will be equal to (P-W^*).

Without further assumption about the functional form of the matching function m, we cannot derive a closed-form solution for the bid-ask spread from Equation (8) and (9). However, our model clearly indicates that such system parameters as(r, λ, P, a, b) all have a significant influence on the magnitude of the bid-ask spread at the market equilibrium. Comparative statics analysis can still be applied on those two equations via implicit function theorem (IFT) to draw many important financial implications.

In particular, if we assume a specific functional form of the matching function as $m(\theta)=\theta^{1/2}$, which is consistent with the increasing and concave properties of m. the entire system can be easily solved since Equation (9) is now independent of W^*:

$$\text{Equation (8)} \rightarrow \quad P-W^*=a(r+\lambda)\theta^{*1/2} \tag{10}$$

$$\text{Equation (9)} \rightarrow \quad \theta^{*1/2}=[(r+\lambda)^2+(P-b)/a]^{1/2}-(r+\lambda) \tag{11}$$

Thus we have Proposition 3.

Proposition 3: *If we assume that the matching function m (θ) between market dealers and investors has a functional form of* $m(\theta)=\theta^{1/2}$*(so m is an increasing and concave function of θ), the prevailing bid-ask spread at the market equilibrium can be expressed by Equation (12):*

$$P-W^*=a(r+\lambda)\{[(r+\lambda)^2+(P-b)/a]^{1/2}-(r+\lambda)\} \tag{12}$$

In the actual security market, there are two closely related but different concepts about the bid-ask spread, one is the quoted bid-ask spread, the other is the effective bid-ask spread. While the quoted bid-ask spread is defined as the difference between the lowest market ask price for a security and the highest market bid price for the same security, the effective bid-ask spread is calculated as twice the difference between the actual execution price and the midquote (the midquote is the average of the market bid and ask price) for a buy order, and twice the difference between the midquote and the actual execution price for a sell order. Here the meaning of "buy" or "sell" is considered from the viewpoint of investors. For instance, a sell order is flowed in if the current quoted market bid and ask prices are \$5.00 per share and \$5.30 per share respectively. The midquote is then (5:00+5:30)/2=\$5.15 per share. Suppose that the market deal steps in front of the previously quoted bid price and the sell order is actually executed at \$5.10, the effective bid-ask spread is 2(5.15 – 5.10) = \$0.10 while the quoted bid-ask spread is 5.30-5.00=\$0.30. Since the effective bid-ask spread better captures the cost of a round-trip order for investors by including the actual execution price in the bid-ask spread calculation, we are going to define the effective bid-ask spread below with the purpose to fit into our theoretical model which only considers the selling part of the security market. When we treat the market equilibrium bid price W^* as the actual execution price and treat the price P as the midquote, the effective bid-ask spread used in our model is defined below:

Definition 2 (Effect bid-ask spread): *The effective bid-ask spread can be express as twice the difference between P and W*. According to Equation (12), the effective bid-ask spread=*

$$2(P-W^*)=2a(r+\lambda)\{[(r+\lambda)^2+(P-b)/a]^{1/2}-(r+\lambda)\} \quad (13)$$

5. Empirical Implications

In this section, we discuss the results of comparative statics analysis of our model, draw important empirical implications and comment the significance and contributions of our competitive search based model.

Basing on the derived bid-ask spread formula, Equation (13), we can clearly see that there are five key parameters (a, b, r, λ, P) which can significantly influence the bid-ask spread $2(P\text{-}W^*)$. In the following, we will discuss in detail the effect of each parameter on the bid-ask spread systemically. Although we assume a simple functional form for the matching function m when deriving our closed form bid-ask spread formula, in fact the impact of the functional form of m on the bid-ask spread should not be ignored, the discussion of which will deserve the work of one full paper and thus has to be omitted here. The only point to be noted is that there may exist multiple equilibriums (Lagos (2007))if the matching function m has the property of increasing returns with respect to its two arguments, v and u, leading to two possible levels of liquidity cost (corresponding to the higher bid-ask spread equilibrium and the lower bid-ask spread equilibrium) for assets with almost identical cash flows(Mandal (2011)) and (Blanchard (1989)).

(1) The effect of the maintenance cost occurring to a market dealer (a) on the bid-ask spread

It has long been known that the maintenance cost occurring to a market dealer is one of the most important factors which can affect the magnitude of the bid-ask spread in the security market. Traditional inventory holding cost theories (Bollena (2004))claim that as the cost of maintaining the role of a market dealer increases, the gap between the bid price and the ask price (i.e. the bid-ask spread) will widen in order to compensate the market dealer for this unavoidable cost. According to

Equation (13), the maintenance cost has two opposite effects: the direct effect of *a* on the bid-ask spread is obviously positive, the indirect effect of *a* on the bid-ask spread is negative. Intuitively speaking, with the increase in *a*, the total number of market dealers ($1-u+v$) in the market should be reduced, either *u* increases or *v* decreases, or both, then the market tightness ($\theta=v/u$), will decrease, which may lead to the possible decrease in the bid-ask spread (Please check Equation (10)). If we assume that the positive direct effect of the maintenance cost dominates the negative indirect one, which is more likely to happen in reality, our model successfully predicts the same result as the conventional inventory holding cost theories do.

(2) The effect of the dividends payoff (*b*) on the bid-ask spread

The effect of the dividends payoff on the bid-ask spread is more straightforward in our model when compared with asymmetric information based models, i.e., there exists a negative relation between the dividends payoff (*b*) and the bid-ask spread. We can explain this relation without difficulty if we think of the dividends payoff as the "negative" maintenance cost to a market dealer, i.e. from the viewpoint of a market dealer, the dividends from holding one share of stock represent some positive carrying benefit. Since the main part of a market dealer's own fund is occupied by the stock inventory holding, the more the amount of dividends paid out to the market dealer, the lower the bid-ask spread required by him or her. While the prediction of our search based model with respect to the effect of the dividends paid out on the bid-ask spread is quite consistent with the hypothesis of asymmetric information based theories(Howe (1992)), both of which are supported by empirical evidence, the underlying story is totally different.

Our model can explain this negative relation without difficulty if we think of the dividends payoff as the "negative" maintenance cost to a market dealer, i.e. from the viewpoint of the market dealer, the dividends from holding one share of the stock represent some positive carrying benefit. Since the main part of the market dealer's own fund is occupied by the stock inventory holding, the more the amount of dividends paid out to the market dealer, the lower the bid-ask spread required by him.

As to the asymmetric information based theories, their underlying hypothesis is that a positive relation exists between the level of information asymmetry and the magnitude of the bid-ask spread. Insofar as the payment of dividends signals material relevant information to the market, thus reducing information asymmetry, dividends policy may influence the bid-ask spread. Moreover, based on the above logic, an inverse relation between dividend yield and bid-ask spread should exist, "ceteris paribus."

Although we believe that our search based explanation is more persuasive than those asymmetric information based stories, whether the negative effect of the dividends payoff on the bid-ask spread originates from our search based market friction or from the asymmetric information friction is more an empirical issue than a theoretical one, the answer of which will be left for further exploration.

(3) The effect of the stock price level (*P*) on the bid-ask spread

Purely looking at the formula of the bid-ask spread in Equation (13), we may draw a conclusion that there exists a monotone positive relation between the stock price level *P* and the bid-ask spread without hesitation. This observation is fully in agreement with Copeland and Galai's model of information effects on the bid-ask spread(Copeland (1983)), i.e. the bid-ask spread is a positive function of the price level. The only difference is that we derive the same result from the perspective of search and matching without the assumption of asymmetric information.

One critical reason why we are greatly interested in the effect of the stock price level on the bid-ask spread is that we are attempting to apply our model to touch on a rather important and prevalent U.S. financial market phenomenon-*stock splits*, and its effect on stock liquidity. As we all know, stock splits are one of the intriguing anomalies in the financial market. Since they only lead to nominal changes in stock prices and there is no any real impact on the equity ownership of a shareholder, stock splits are not supposed to have any material effect on the stock price behavior and the measure of liquidity subsequent to the splits though the opposite is true in reality.

Referring to the effect of stock splits on the bid-ask spread, two strands of theories are competing with each other. The liquidity and trading range hypothesis claims that the motivation for stock splits is to bring the stock price down to a preferred trading range in order to improve its liquidity. This hypothesis is strongly supported by management in practice because most managers who are in charge of stock splits decisions do believe that the above consideration is indeed the dominating concern of their decisions on stock splits. When the bid-ask spread is utilized as our measure of liquidity from now on, the accompanying financial implication is that the bid-ask spread should decrease after stock splits, i.e. *improved liquidity follows stock splits.*

Alternative asymmetric information based theory proposed by Conroy, Harris, and Benet (Conroy (1990))suggests that stock splits with the feature or function of worsening liquidity can serve as a costly but valid signal of "favorable future prospects of the firm". The corresponding implication is that the bid-ask spread should increase after stock splits, i.e. *worsen liquidity follows stock splits*.

When resorting to empirical research to appraise the validity of those two rival theories, the existing empirical results about the impact of stock splits on liquidity are mixed as well. The inconclusive evidence partly reflects the challenge in selecting and interpreting the proper proxy for the measure of liquidity. While the liquidity and trading range hypothesis selects the absolute bid-ask spread in dollars (such as $2(P\text{-}W^*)$ in our model) as its measure of liquidity, the asymmetric information based theory prefers to applying the relative bid-ask spread in percentage (such as $2(P\text{-}W^*)/P$ in our model). Each theory uses the corresponding empirical results to support its own claim on the relation between stock splits and liquidity. For instance, Conroy, Harris, and Benet in the same paper find that "percentage spreads increase after splits, representing a liquidity cost to investors" for NYSE listed companies.

Summarizing the existing conflicting empirical results, on the one side, the absolute bid-ask spread is positively related to the stock price level, on the other side, the relative bid-ask spread is negatively related to the stock price. Our competitive search based model can solve this apparently controversial issue elegantly:

Firstly, consider Equation (13), let all the other parameters fixed, when P decreases (b also decrease in the same proportion during this process), the absolute bid-ask spread will decrease thereafter, showing that the absolute bid-ask spread is positively related to the stock price level P;

Secondly, divide both sides of Equation (13) by P in order to obtain the formula for the relative bid-ask spread. Roughly speaking, the numerator of the right-hand side of Equation (13) has the power of ½ for P, the denominator has the power of 1 for P, and then the relative bid-ask spread has the power of -1/2 for P. Thus, the relative bid-ask spread is approximately a decreasing function of the stock price level P.

(4) The effect of the discount rate (*r*) on the bid-ask spread

The effect of the (risk-free) discount rate on the bid-ask spread is roughly positive since in most of times, the positive effect of the discount rate on the bid-ask spread dominates its negative effect. It is not difficult to understand this positive effect if we think of the discount rate as the opportunity cost of holding the stock shares without earning the interest for the market dealer's own fund. The higher the discount rate is, the more the amount of the interest given up by the market dealer when he or she keeps an inventory of stocks, thus the higher bid-ask spread will be required for compensation.

(5)The effect of the instantaneous market opportunity (λ) on the bid-ask spread

λ is unique to our competitive search based model. While the effect of λ on the bid-ask spread is very similar to that of the discount rate r since both parameters show up in the same position in our bid-ask spread formula, the underlying financial meaning of λ is rather subtle and thus sensitive to explanation.

Formally, λ is defined as the market dealer's cancellation rate for an existing deal, i.e. when the market dealer expects that the posted bid price, W^* may not be appropriate for the on-going market sentiment or financial situation, the market dealer will cancel it. From this viewpoint, we image λ as the measure of the instantaneous market opportunity.

Alternatively, since either too low or too high W^* is equally likely to cause the market dealer to cancel the existing deal, λ can also be treated as the measure of the current overall market uncertainty because in our model we have only one type of asset (or stock) traded in the entire security market. Higher λ means that the market dealer feels that there exists more uncertainty in the market and thus it is more likely for a posted bid price W^* unfit for the current market condition, leading to a deal cancelled more often.

In short, we summarize our findings in Proposition 4:

Proposition 4: (Determinants of the bid-ask spread)

(1) The bid-ask spread is positively related to the market dealer's maintenance cost a, the discount rate r and the instantaneous market opportunity λ;

(2) The bid-ask spread is negatively related to the dividends from holding one share b.

(3) The bid-ask spread is positively related to the stock price level P; but the percentage bid-ask spread is negatively related to the stock price level P.

Moreover, there are the other two important results which can be derived from our model:

(A) Quantification of the relative importance of the search and matching friction cost and the inventory-holding cost.

The relative weight of the inventory-holding cost and the search and matching cost is basically an empirical issue, on which our model has the potential to shed light. Basing on our bid-ask spread formula in Equation (13), though the two terms are coupled with each other in our model, if the inventory-holding cost is roughly proxied by the maintenance cost a, the search and matching cost is then estimated by $(P-W^*-a)$ Thus the ratio of the two types of costs will be $(P-W^*-a)/a$.

(B) Determination of the number of market dealers supported by the market structure.

Another interesting result is related to the number of market dealers which can be supported by the security market. Since the market tightness $\theta^* = v/u$, if u is known, we can pin down the total number of market dealers at the market equilibrium, which is equal to $(1-u+v)$, here, $1-u$ is the number of market dealers who have business, v is the number of market dealers who are idle.

One salient feature of our model is that we don't assume the total number of market dealers be fixed, ex ante. This number is endogenously decided by the system via a free entry and exit condition for market dealers, which is the reason why our model is called the "competitive" search model. Intuitively, some researchers give a pre-emptive monopoly position to one market dealer. Therefore their models may lead to a comparatively higher bid-ask spread owing to the monopoly profit.

6. Calibration and simulation

In this section, we calibrate the key parameters of our bid-ask spread formula according to the typical values of the stock market in order to show the impacts of those parameters on the bid-ask spread quantitatively. Our simulation results show that the bid-ask spread indicated by our competitive search based model well fits into the empirical observations. However, it should be noted that the choices of values of model parameters may have an important effect on the magnitude of the bid-ask spread.

Table 1 summarizes the key parameters and their typical values which will be used in our model simulation.

Table 1
Parameters and Their Values Used in the Model

PARAMETER	NOTATION	TYPICAL VALUE
Maintenance Cost Occurring To A Market Dealer	a	\$0.001
Dividends Produced Each Period By One Share	b	\$1.000
Stock Price Level	P	\$60.000
Discount Rate	r	0.040
Instantaneous Market Opportunity	λ	1.000

Fig. I The effect of the maintenance cost on the bid-ask spread

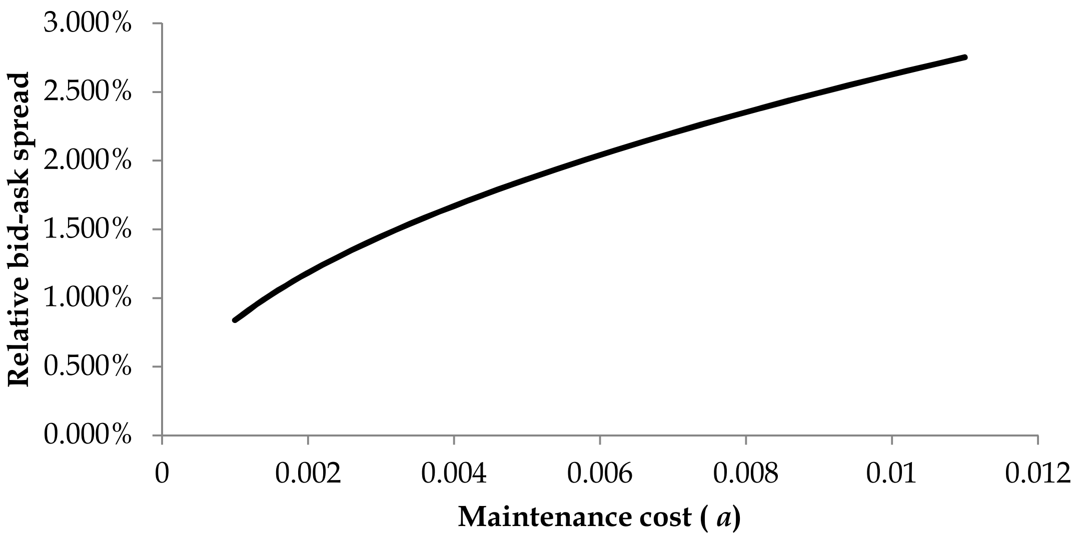


Figure I shows the positive effect of the maintenance cost (a) on the relative bid-ask spread ($2(P\text{-}W^*)/P$) when the values of the other parameters from Table I are fixed. We let the maintenance cost have about 10 times fluctuations from the starting value of \$0.001 per period to \$0.011 per period.

We use the recently average stock price of S&P 500 companies, \$60 per share as our typical stock price. Considering the average dividend yield for industrial stocks in the S&P 500 is about 2%, we normalize the dividends produced during each period by one share of the stock to \$1. We also set the maintenance cost occurring to the market dealer to be tiny, i.e. \$0.001 per period, in order to stress the importance of the role played by the market search friction on the bid-ask spread. We choose the median of monthly 10-year Treasury constant maturities rates from January, 2000 to December, 2014, which is approximately 0.04 (i.e. 4%) as the discount rate. The instantaneous market opportunity represents the overall market uncertainty and is not an easy task to estimate its numeral value. Since the managed mutual funds have an average turnover rate of approximately 85%, we use 1(i.e. 100%) as the instantaneous market opportunity arrival rate.

Fig. II The effect of the dividend yield on the bid-ask spread

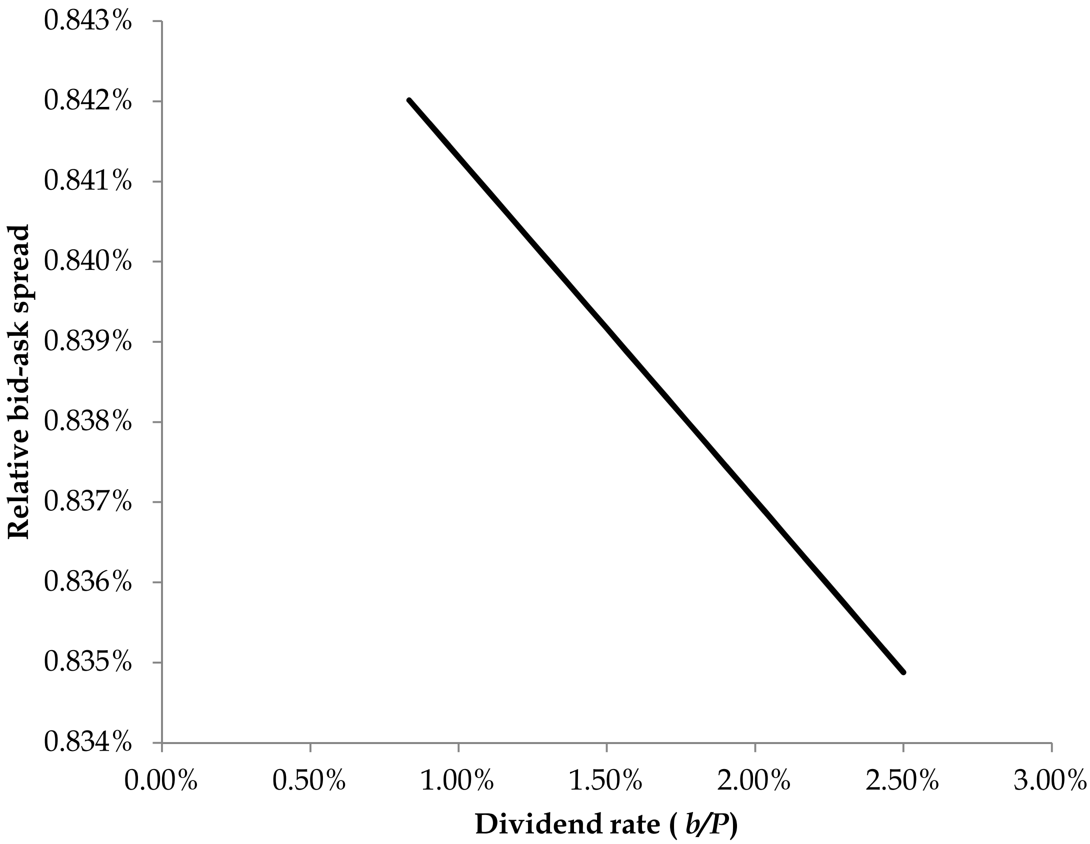


Figure II shows the negative effect of the dividend yield (*b/P*) on the bid-ask spread. The possible value of the dividends produced during each period by one share of the stock ranges from \$0.5 to \$1.5. In another word, when the stock price *P* =\$60, the corresponding dividend yield spans from 0.83% (i.e. 0.5/60) to 2.5% (i.e. 1.5/60).

Fig. III(a) The effect of the stock price level on the relative bid-ask spread

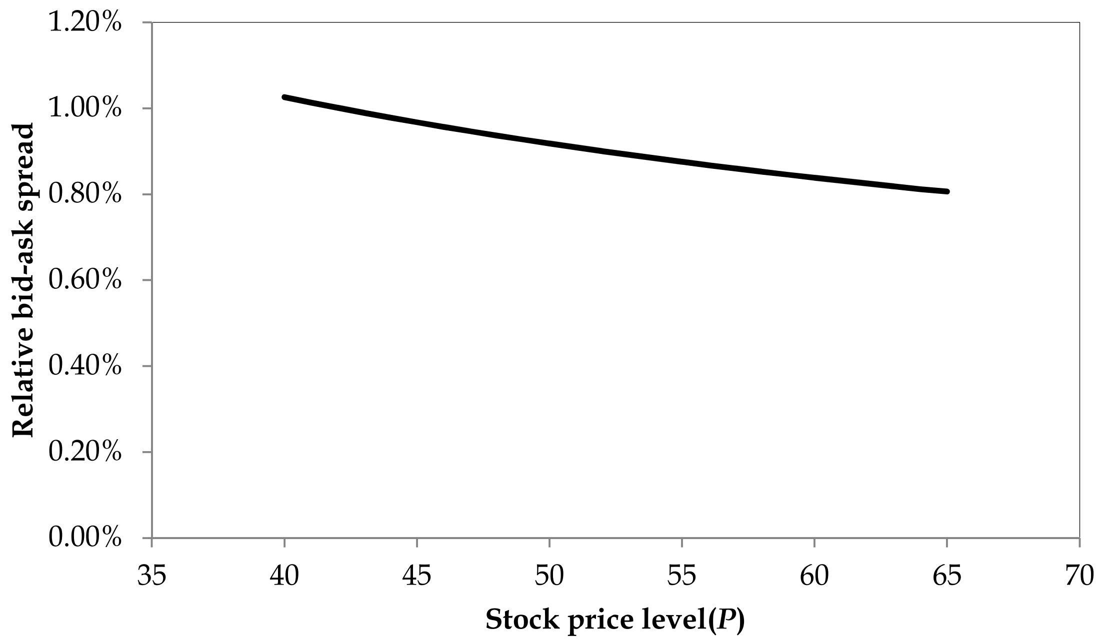


Fig. III(b) The effect of the stock price level on the absolute bid-ask spread

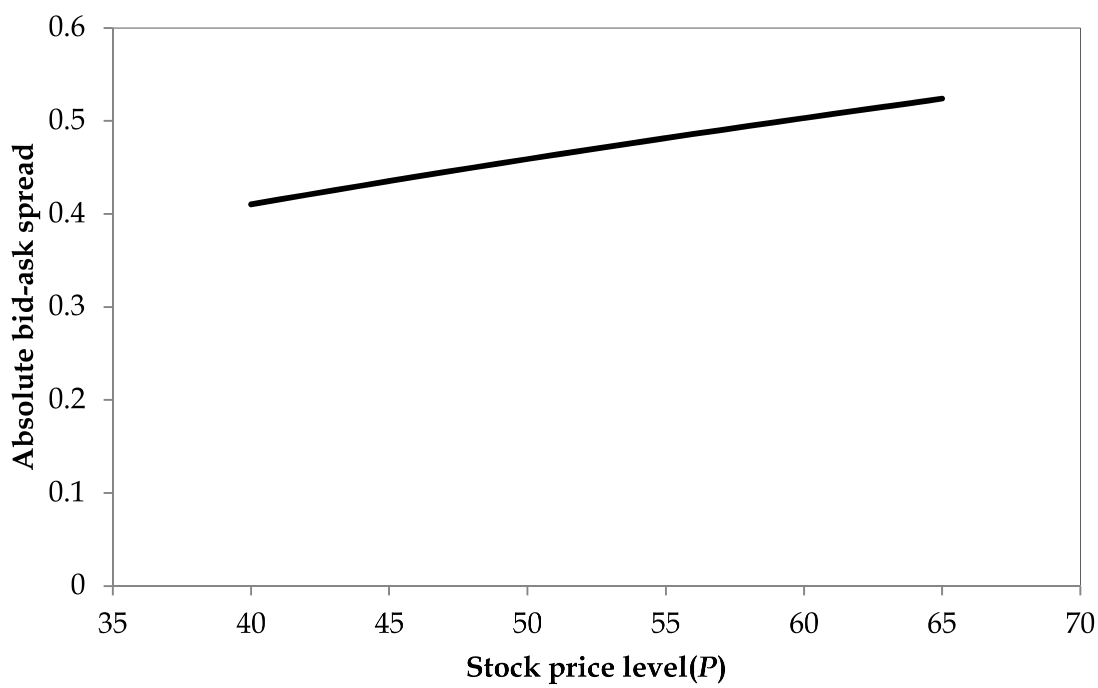
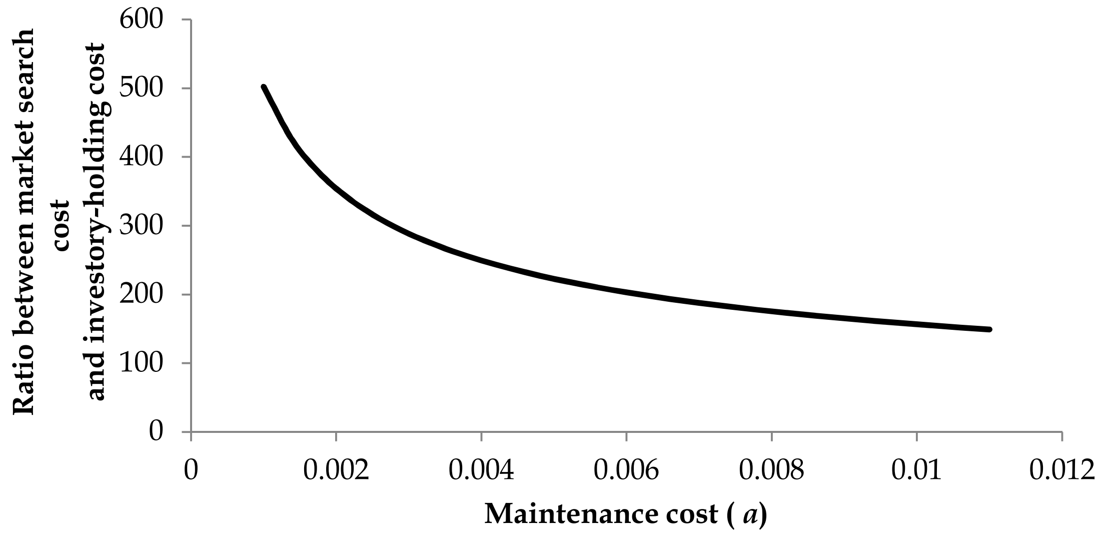


Figure III (a) and (b) show the effects of the stock price level (P) on both the relative (in percentage) bid-ask spread (Figure III(a)) and the absolute (in dollars) bid-ask spread (Figure III(b)) when the typical values of the other parameters are used from Table I except that we keep the dividends payoff b as a constant ratio of the stock price P, i.e. $b=(1/60)P$ because when there is a stock split, the corresponding dividend should also be split. The possible value of the stock price level ranges from $40 to $65. We can see clearly that the relative/percentage bid-ask spread is negatively related to the stock price level P while the absolute /dollar bid-ask spread is positively related to the stock price level P.

Fig. IV Decomposition of the bid-ask spread



Moreover, we decompose the bid-ask spread into two parts: the inventory –holding cost which can be approximated by a in our model, and the residual part will be the market search and matching cost. We can then evaluate the relative weight of the inventory-holding cost and the market search and matching cost showed in Figure IV where with the increase of the maintenance cost the market search and matching cost becomes less important but the former is still fully dominate the latter in magnitude under our model structure.

7. Conclusion

In this paper, the optimal behaviors of both market dealers and investors are simultaneously investigated under the framework of competitive search theory. Four useful value functions for both agents are established to represent the corresponding utilities obtained when staying in two distinct phases, the idle phase and the occupied phase.

The model shows that the bid-ask spread charged by the market dealer is legitimated. The magnitude of the bid-ask spread is determined by many factors such as the cost to maintain the market dealer's position, the (risk-free) discount rate, the fluctuation of market opportunity and the tightness of market condition, etc.

Furthermore, the model's prediction on the effect of dividend policy on the bid-ask spread is consistent with asymmetric information based theories and empirical results. More importantly, the model makes a distinction between the absolute bid-ask spread and the relative bid-ask spread and thus resolves the apparent controversial issue on the effect of stock splits on stock liquidity.

Different from traditional bid-ask spread theories which pay much attention to the inventory-holding cost and the asymmetric information cost associated with two types of investors in the market, our competitive search model stresses the importance of the market search friction.

It has to be admitted that with the rapid progress of advanced information technology and more centralized security transactions, searching for the counter party has become more efficient. Thus the market search friction plays a less important role for the operations of security markets than previously. We, however, can still appreciate the merits of our search based model from two aspects:

Firstly, well-organized central markets historically often evolve from the initial disordered and de-centralized markets. When considering the compensation for the role of market dealers playing in a centralized market in the form of the bid-ask spread, we need to compare two states, one is the actual centralized market, and the other is the imaginary de-centralized market, even if the de-centralized market is not the current status of the market structure. To be more specific, only if we have explored how difficult it is to meet the other side of the transaction in the imaginary de-centralized market, can we then evaluate accurately how well the actual centralized market provides the liquidity to investors and justify the amount of the bid-ask spread charged by market dealers in the current centralized market.

Secondly, our competitive search based model well characterizes the matching process between market dealers and investors. As we have mentioned before, our competitive based model is distinguished from traditional random search based models by how the search process proceeds. While random search based models assume 1) the random matching between two types of agents, and 2) the determination of price by bargaining once they meet, neither of which fits into the reality of a typical security market, our competitive search based model allows market dealers to post a widely known price quote in public ex ante in order to direct or attract the arrival of investors. Furthermore, our concept of symmetric equilibrium lets all market dealers play the same strategy at the market equilibrium, i.e. post the same price quote in the entire market. In this way, our model mimics the operation of securities markets better than traditional random based search models.

The existence of market dealers(or any other agent with the same responsibility but the different title in a security market) is so indispensible and so natural that it is hard to image what would happen if there were not such a role there. As we never take a pause to challenge why the New York Stock Exchange charges a variety of service fees for securities trading due to the secondary market liquidity provided to the entire society by it, so in the same logic any reasonable explanation of why the bid-ask spread is charged by market dealers for security transactions should be traced back to the service offered by them. The role played by asymmetric information in the bid-ask spread phenomenon matters only to some extent via modulating the above fundamentals, which is the basic belief of the authors and the motivation of this paper as well.

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Appendix A

Notation Table

f**:** market dealer.
w**:** investors.
P**:** the instantaneous price at which a market dealer can dispose of one share of stock. (Since we only model the selling part of the market, *P* can be treated as parameter.)
W**:** the instantaneous bid price posted by a market dealer.
P-W**:** the price difference between *P* and *W*, denoting the bid-ask spread.
1: the initial measure of all investors normalized to 1.
u**:** the measure of investors who don't sell their shares.
1-*u*: the measure of investors who sell their shares, which is also the measure of market dealers who have business.
v**:** the measure of market dealers who are idle.
a**:** the maintenance cost occurring to a market dealer when posting a bid price in the market.
b**:** the units of dividends produced each period by one share as one Lucas tree.
θ**:** measures the market tightness, equals $\frac{v}{u}$.
$m(\theta)$**:** the matching technology function between *f* and *w*, which is an increasing (and concave) function of θ. Thus, the arrival rate for an investor is $m(\theta)$, for a market dealer is $m(\theta)/\theta$, respectively.
λ**:** the instantaneous market opportunity, follows a standard Poisson distribution.
r**:** the discount rate
State "*U*": means that it is in the idle state.
State "*V*": means that it is in the occupied state.
U_f : the value of a market dealer who posts a bid price and waits for a business.
V_f : the value of a market dealer who buys one share from an investor.
U_w: the value of an investor who keeps one share in hand and waits for an chance to sell.
V_w : the value of an investor who sells one share to a market dealer.

Appendix B

Proofs of Propositions

Proposition 1: Market equilibrium (Equivalence of two optimal problems: (6) and **(7))**

From the optimal problem (6),

$$L_1 = U_f\,(W,\theta) + \mu\,[U_w(W,\theta\,) - U_w(W^*,\theta^*)]$$

F.O.C. for W: $\frac{\partial L_1}{\partial W} = \frac{\partial U_f}{\partial W} + \mu \frac{\partial U_w}{\partial W} = 0$

for θ: $\frac{\partial L_1}{\partial \theta} = \frac{\partial U_f}{\partial \theta} + \mu \frac{\partial U_w}{\partial \theta} = 0$

From the optimal problem (7),

$$L_2 = U_w(W,\theta) + \eta\, U_f(W,\theta\,)$$

F.O.C. for W: $\frac{\partial L_2}{\partial W} = \frac{\partial U_w}{\partial W} + \eta \frac{\partial U_f}{\partial W} = 0$

for θ: $\frac{\partial L_2}{\partial \theta} = \frac{\partial U_w}{\partial \theta} + \eta \frac{\partial U_f}{\partial \theta} = 0$

When set μ=$1/\eta$, those two sets of conditions are equivalent with each other.

Proposition 2: Two equilibrium equations

(1) Derive the free entry equation:

Step one: Solve for U_f.

For a market dealer, (3)-(4), we get:

$r\,(V_f - U_f) = P - W + a - (V_f - U_f)(\lambda + m/\theta)$

$(V_f - U_f) = (P - W + a)/(r + \lambda + m/\theta)$

Put the above relation back into (4),

$$\begin{aligned} rU_f &= -a + (m/\,\theta)(V_f - U_f) = -a + \,(m/\,\theta)[(P - W + a)/(r + \lambda + m/\theta)] \\ &= -a + [m(P - W + a)]/[\,\theta(r + \,\lambda) + m] \\ &= [m(P - W) - a\,\theta(r + \,\lambda)]/\,[\,\theta(r + \,\lambda) + m] \end{aligned}$$

So $U_f = \,[m(P - W) - a\,\theta(r + \,\lambda)]/\,[\,r\theta(r + \,\lambda) + rm]$

Step two: Use (5), since $U_f = 0$, the numerator has to be zero,

i.e. $m(P - W) - a\,\theta(r + \,\lambda) = 0$

At market equilibrium, W^* and θ^*, we get:

$m(\theta^*)(P - W^*) \,-\, a\,(r + \lambda)\theta^* = 0$

(2) Derive the Nash equilibrium equation:

Step one: Solve for U_f.

According to the result derived from the free entry equation, we have known that:

$U_f = \,[m(P - W) - a\,\theta(r + \,\lambda)]/\,[\,r\theta(r + \,\lambda) + rm]$

Step two: Solve for U_w.

For an investor, (2)-(1), we get:

$r\,(V_w - U_w) = W - b - (V_w - U_w)(\lambda + m)$

$(V_w - U_w) = (W - b)/(r + \lambda + m)$

Put the above relation back into (1),

$$\begin{aligned} rU_w &= b + m(V_w - U_w) = b + \,m[(W - b)/(r + \lambda + m)] \\ &= [mW + (r + \,\lambda)b]/\,(r + \,\lambda + m) \end{aligned}$$

So $U_w = [mW + (r + \,\lambda)b]/\,[r(r + \,\lambda + m)]$

Step three: Solve the optimal problem (6)

Use the LaGrange method, set up two Lagrange multipliers μ and η since there are two constraints (actually the second constraint has no effect on the first order conditions).

$L = U_f(W,\theta) - \mu\,[U_w(W,\theta) - U_w(W^*,\theta^*)] - \eta[U_f(W^*,\theta^*)]$

Put $U_f(W, \theta)$ and $U_w(W, \theta)$ derived from Step one and Step two into L, we get:

$$L = [m(P - W) - a\,\theta(r + \lambda)]/\,[\,r\theta(r + \lambda) + rm] - \mu[rU_w^*(r + \lambda + m) - mW - (r + \lambda)b] - \eta[U_f(W^*,\theta^*)]$$

Here, $U_w{}^*$ is the abbreviated form of $U_w(W^*, \theta^*)$.

Now consider:

First order condition for W:

$-m/[\,r\theta(r + \lambda) + rm] + \mu m = 0$

So, $\mu = 1/[\,r\theta(r + \lambda) + rm] = 1/\{r[\theta(r + \lambda) + m]\}$

First order condition for θ:

(Note that m is also a function of θ)

$$(1/r)\,\{[m'(P - W) - a(r + \lambda)][\,\theta(r + \lambda) + m] - [m(P - W) - a\,\theta(r + \lambda)]\,[\,(r + \lambda) + m']\}/[\theta(r + \lambda) + m]^2 - \mu(rU_w^* m' - m'W) = 0$$

Then put $\mu = 1/\{r[\theta(r + \lambda) + m]\}$ into the above equation, delete the factor of $1/r$ and $1/[\theta(r + \lambda) + m]$ on both items,

$$[m'(P - W) - a(r + \lambda)] - \{[m(P - W) - a\,\theta(r + \lambda)]\,[\,(r + \lambda) + m']\}/[\theta(r + \lambda) + m] - (rU_w^* m' - m'W) = 0$$

Apply the free entry condition, at market equilibrium (W^*, θ^*), we know that:

$m(\theta^*)(P - W^*) - a\,\theta^*(r + \lambda) = 0$, So the middle item is equal to zero.

Finally only the first and last items are left:

$[m'(P - W) - a(r + \lambda)] - (rU_w^* m' - m'W) = 0$

$m'(P - W) - a(r + \lambda) - m'(rU_w^* - W) = 0$

$m'(P - rU_w^*) - a(r + \lambda) = 0$

Moreover, replace $rU_w{}^*$ by $r\,U_w(W^*,\theta^*) = [mW^* + (r + \lambda)b]/\,(r + \lambda + m)$,

$m'[P(r + \lambda + m) - mW^* - (r + \lambda)b] - a(r + \lambda)\,(r + \lambda + m) = 0$

$m'[P(r + \lambda) + mP - mW^* - (r + \lambda)b] - a(r + \lambda)\,(r + \lambda + m) = 0$

According to the free entry condition:

$m(\theta^*)(P - W^*) - a\,\theta^*(r + \lambda) = 0$

so $mP - mW^* = a\,\theta^*(r + \lambda)$,

Thus, at market equilibrium, the F.O.C. for θ changes into:

$m'[P(r + \lambda) + a\,\theta^*(r + \lambda) - (r + \lambda)b] - a(r + \lambda)\,(r + \lambda + m) = 0$

$m'(P + a\,\theta^* - b) - a(r + \lambda + m) = 0$

i.e. $m'(\theta^*)(P - b) - a[r + \lambda + m(\theta^*) - \theta^*\,m'(\theta^*)] = 0$

Proposition 3: Bid-ask spread formula

This proposition has already been proved in the paper. The brief proof is reproduced here for your reference.

If $m(\theta) = \theta^{1/2}$,

The free entry equation changes into: $\theta^{*1/2}(P - W^*) - a(r + \lambda)\,\theta^* = 0$

i.e. $P - W^* = a(r + \lambda)\,\theta^{*1/2}$

The Nash equilibrium equation changes into:

$0.5\,\theta^{*-1/2}(P-b) - a(r+\ \lambda +\ \theta^{*1/2} - 0.5\,\theta^{*}\,\theta^{*-1/2})\ = 0$

$0.5\,\theta^{*-1/2}(P-b) - a(r+\ \lambda +\ 0.5\theta^{*1/2})\ = 0$

Let $\theta^{*1/2} = X$, then, $\theta^{*-1/2} = 1/X$, the above equation can be changed into:

$X^2 + 2(r+\ \lambda)X - (P-b)/a = 0$

Solve the above quadratic equation, get X,

$\theta^{*1/2} = X = [(r+\lambda)^2 + (P-b)/a]^{1/2} - (r+\lambda)$

Thus, the bid-ask spread is:

$P - W^{*} = a(r+\lambda)\,\theta^{*1/2} =\ a(r+\lambda)\,\{[(r+\lambda)^2 + (P-b)/a]^{1/2} - (r+\lambda)\}$

Proposition 4: Determinants of bid-ask spread

It is easy to check out the signs of the first derivations of our bid-ask spread on those determinants.