Value at Risk under Jump GARCH Processes

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VaR is central to financial risk management. In this study we use three methods to calculate VaR, namely the industry standard RiskMetrics, Monte Carlo simulation and Filtered Historical simulation. To model the return we use GARCH and NGARCH processes augmented with normally distributed jumps. We examine the implication for In-Sample testing in the presence of jumps and find that the inclusion of jumps generally leads to fewer VaR violations. This finding is consistent with the findings of other researchers. However, at higher confidence intervals and long horizon dates all methods and models fail In-Sample testing.
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1. Introduction

Over the years a whole set of measures have been developed to summarise the riskiness of the return on an individual asset or a portfolio of assets. In today's financial institutions, *VaR* (Value at Risk) has become by far the most widely used measure of risk. *VaR* is defined as the loss that will be exceeded over a certain holding period such as a day, 10 days etc on a defined fraction of events, say 1%.Thus estimating *VaR* is equivalent to estimating the quantiles of the distribution of returns.

Value at risk may be calculated using the non-parametric historical simulation approach which uses past data as an indication to what may occur in the future. Alternatively it may be calculated using the parametric model building approach in which the asset returns follow a statistical model.

In this study we focus on the parametric approach to *VaR* estimation. We use the RiskMetrics approach, Monte Carlo simulation and Filtered Historical simulation. All three methods rely on modeling the volatility as a means to generating the *VaR*. The RiskMetrics model forecasts tomorrows' volatility as a weighted average of today's volatility and today's squared return. The advantages of the RiskMetrics approach are simplicity and that it is able to track variance changes in a way that is broadly consistent with observed returns. The disadvantages are that it is not able to cater for variance clustering and provides counterfactual longer horizon forecasts.

To overcome the weaknesses of variance clustering and counterfactual forecasts Monte Carlo simulation and Filtered Historical simulation are used. These two approaches overcome the variance clustering issue by parametrically modeling the return process and the *K* day horizon issue by simulating *K* days into the future under different scenarios. Thus the key difference between RiskMetrics and the two simulation approach is the way the volatility is modeled. Given that the volatility is modeled using GARCH type process in the latter cases, in theory the simulation approach should be more accurate than the RiskMetrics. The key difference between Monte Carlo simulation and Filtered Historical simulation is the way the random numbers are generated. In the Monte Carlo approach the random numbers are generated by the software, whereas in the case of Filtered Historical simulation random numbers are taken from a database based on the standardised residuals of returns based on the asset in consideration from a recent period. As such these standardised residuals should be more representative of the recent experience and hence the Filtered Historical simulation should lead to more accurate *VaR* than standard Monte Carlo simulation. One of the main objectives of the paper is to check whether Monte Carlo simulation is more accurate than RiskMetrics and if Filtered Historical simulation is more accurate than Monte Carlo simulation particularly as the horizon and confidence level increases.

Simple statistical models impose strong conditions on the characteristics of the data. For example, imposing a normal density function on the asset returns implies that the underlying asset return has constant mean and variance in sharp contrast to empirical evidence which shows that financial time series have certain common characteristics. First volatility clustering is often observed as noted by Mandelbrot (1963).This feature refers to the fact that large changes tend to be followed by large changes and small changes tend to be followed by small changes. Second, leptokurtosis is observed, i.e., the kurtosis exceeds the kurtosis of a standard Gaussian distribution. Third the leverage effect noted by Black (1976), where a negative return increases variance by more than a positive return of the same magnitude. In order to account for these time series characteristics, volatility may be modelled as one or two shock process.

In a path breaking paper, Engle (1982) proposed to model time-varying conditional variance with Autoregressive Conditional Heteroskedasticity (ARCH) process, which uses past disturbances to model the variance of the process. The ARCH process was further generalised by Bollerslev (1986) to GARCH. Both the ARCH and the GARCH fail to take into account the leverage effect. To incorporate this feature, large numbers of alternative GARCH specifications have been proposed such the NGARCH (non-linear GARCH), EGARCH (exponential GARCH) by Nelson (1991) amongst others. The stochastic volatility models allow the volatility to be driven randomly by an unobserved process. There are a wide variety of stochastic volatility models in the literature (for example, Hull and White (87), Wiggins (87), Heston (93)).Despite their sophistication both GARCH and stochastic volatility models are unable to incorporate the rapid changes that may occur in the event of a stock market crash such as 1987, or the more recent stock market crash due to the credit crunch. As a result a number of studies have attempted to incorporate rapid jumps along side time varying volatility. Examples include the study by Bates (1996) for foreign exchange rates with stochastic volatility model and interest rates by Das (2002) for ARCH models. Indeed, a number of researchers including Venkataraman (1997),Zangari (1996,1997) argue that jumps improve on standard VaR calculations used in risk management.

In this study we focus solely on returns augmented with jumps. In particular we augment the GARCH and the NGARCH with Poisson type jump processes. We assume that the jumps are normally distributed with a non-zero mean and finite variance. We examine the impact of jumps on VaR calculations over multiple horizons. By performing In-Sample testing this paper shows how incorporating jumps has a significant impact on VaR. We focus on the period starting from October 1985 and ending in 2003 as this period contains the 1987 crash.

2. Parameter Estimation and Value at Risk

Empirically it is observed that the unconditional distribution o daily returns has a fatter tail than the normal distribution; equity and equity indices returns displays negative correlation with the variance as measured by the square of the returns; high period of volatility over time will revert back to a historical average volatility.GARCH(1,1) and its more complex variants incorporate all of these observed features. However, even after standardising returns by a time varying volatility measure, they still have a fatter tail than normal tails. In other words, GARCH models are not capable of adequately modelling the kurtosis or tail fatness observed in equity and equity index returns. To model the excess kurtosis, jumps are introduced both to the returns process and the volatility process.

Given that the mean of daily return is very small we take the general asset return process as given by

$$R_t = \sigma_t \varepsilon_t \tag{1}$$

where $\varepsilon_t \mid \Psi_{t-1} \sim N\left(0,1\right)$, and Ψ_{t-1}_{t-1} is the time t-1 information set and σ_t^2 is the conditional volatility.

There are many different ways of specifying the conditional volatility σ_t. The industry standard as advocated by JP Morgan's RiskMetrics sets the conditional mean constant, and specifies the variance as:

$$\sigma_t^2 = (1-\lambda)\varepsilon_{t-1}^2 + \lambda\sigma_{t-1}^2 \tag{2}$$

where λ is set to 0.94 for daily data and the innovations are Gaussian. An alternative specification of the conditional volatility is the GARCH(1,1) (Bollerslev, 1986) where the conditional variance evolves as

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3}$$

The above specification ignores the leverage effect, thus a modified version, the NGARCH(1,1) is:

$$\sigma_t^2 = \omega + \alpha\left(R_{t-1} - \theta\sigma_{t-1}\right)^2 + \beta\sigma_{t-1}^2 \tag{4}$$

For the remaining part of this study we add a random jump, specifics of which is to be defined later

$$R_t = \sigma_t\varepsilon_t + \text{Jump} \tag{5}$$

where σ_t is either GARCH(1,1) or NGARCH(1,1) process.

2.1. Parameter Estimation

There is a growing body of literature on the estimation of jump diffusion processes. Ball and Torus (1983) and Jorion (1988) find evidence for jumps in the equity and foreign exchange markets and Bates (1996) carries out an extensive study of equity and forex option markets for combined jump and stochastic volatility models. Das (2002) find evidence for jumps in interest rates.

We estimate the Poisson Gaussian return process using a Bernoulli approximation first introduced in Ball and Torus (1983).They made the assumption that in each time interval either only one jump occurs or no jump occurs. Given that we are using short frequency data, this assumption is reasonable. Ball and Torus (1983) found that this approximation provides an estimation procedure that is highly tractable, stable and convergent. Given that the limit of the Bernoulli process is governed by a Poisson distribution, we can approximate the likelihood function for the Poisson Gaussian model using a combination of the normal distribution dictating the diffusion and jump shocks. The asset return incorporating jumps is given by:

$$R_t = \sigma_t\varepsilon_t + J\left(\mu, \gamma^2\right)\Delta\pi\left(q\right) \tag{6}$$

where $J\left(\mu, \gamma^2\right)$ is the jump shock, which is normally distributed with mean μ and variance γ^2. $\Delta\pi\left(q\right)$ is the discrete time Poisson increment, approximated by a Bernoulli distribution with jump intensity q.

The variance σ_t^2 is either a GARCH(1,1) or NGARCH(1,1). The transition probabilities for the asset return following a Poisson-Gaussian process can be stated as

$$f\left(R_t\right) = \frac{q}{\sqrt{2\pi\left(\sigma_t^2 + \gamma^2\right)}}\exp\left(-\frac{\left(R_t - \mu\right)^2}{2\left(\sigma_t^2 + \gamma^2\right)}\right) + \frac{1-q}{\sqrt{2\pi\sigma_t^2}}\exp\left(-\frac{R_t^2}{2\sigma_t^2}\right) \tag{7}$$

The above density function approximates the true Poisson-Gaussian density function, as it assumes the possibility of a single jump at most per period with a mixture of normal distributions. Using the above density function we are unable to locate the precise timing of the jumps, however the above density function can be used as the basis of parameter estimation using the likelihood function L for a total of T observations

$$L = \prod_{t=1}^{T} f\left(R_t\right) \tag{8}$$

which may be stated using natural logarithm as:

$$\max_{\alpha,\beta,\theta,\omega,\mu,\gamma^2,q} \sum_{t=1}^{T} \ln\left(f\left(R_t\right)\right) \tag{9}$$

We use the above mixture of distribution to obtain the quasi maximum-likelihood estimate of the parameters subject to certain constraints. The constraints are that the weights in each regime (jump vs. no-jump) sum to one and 0≤q≤1 which is imposed during estimation. Quasi MLE is obtained as a solution to a system of first order equations $\partial \ln L / \partial \Omega$ equal zero, where L is the pseudo-likelihood function.

2.2. Value at Risk and In-Sample Testing

Value at Risk is a conditional quantile of the loss distribution. The VaR measure for time *t* with coverage probability *p* is based on information at time *t*-1, and is defined as the positive value VaR_t^p such that

$$\Pr\left(-R_t > VaR_t^p\right) = p \tag{10}$$

Usually p is a small number such as 1% or 5%. Using equation (10) we can further simplify the expression VaR_t^p. Specifically:

$$VaR_t^p = \sigma_t G_p^{-1} = \sigma_t c_p \tag{11}$$

where G_p^{-1} denotes the (1-*p*)th quantile of *G* and the distribution of standardised losses $\varepsilon_t = R_t / \sigma_t$. Thus if *G* is the standard normal distribution with *p* = 0.05, then $G_p^{-1} = \Phi_{0.95}^{-1} = 1.65$, hence $VaR_t^p = 1.645\sigma_t$. More generally where $\varepsilon \sim G$, we can state VaR_t^p as a product of σ_t with constant $c_p = G_{1-p}^{-1}$ whose value depends on *p* and *G*.

In this paper we consider three ways of estimating Value at Risk. The first is the industry standard RiskMetrics where the volatility is specified by equation (2). The second approach involves Monte Carlo simulation to produce a series of hypothetical returns based on an assumed distribution from day *t*+1 to day *t*+*K*. Based on these hypothetical returns, we calculate the hypothetical K-day return for each Monte Carlo path. Thus, if we perform *N* Monte Carlo simulation

$$\hat{R}_{i,t+1:t+K} = \sum_{k=1}^{K} \tilde{R}_{i,t+k} \text{for } i = 1,2,\ldots,N \tag{12}$$

Collecting the *N* hypothetical *K*-day returns in a set $\left\{\hat{R}_{i,t+1:t+k}\right\}_{i=1}^{N}$ allows us to calculate the *K*- day VaR by calculating the 100*p* percentile

$$VaR_{t+1:t+K}^p = -\text{Percentile}\left\{\left\{\hat{R}_{i,t+1:t+k}\right\}_{i=1}^{N}, 100p\right\} \tag{13}$$

The final approach proposed by Barone-Adesi et al (1999, 2002) used is the Filtered Historical Simulation (FHS) approach which combines a model-based variance with historical simulation - which relies solely on empirical distribution based on past losses. Specifically given a sequence of past returns, $\left\{R_{t+1-\tau}\right\}_{\tau=1}^{m}$ estimate a set of standardised returns $\left\{z_{t+1-\tau}\right\}_{\tau=1}^{m}$. The key difference between the standard Monte Carlo approach and this approach is that with FHS we use the residual error terms to generate the future hypothetical returns rather than the random numbers based on an assumed distribution.

Based on a time series of past ex ante VaR forecasts and past ex post returns, we define the hit sequence of VaR exceedances as:

$$I_{t+1} = \begin{cases} 1 \text{ if } R_{PF,t+1} < -VaR_{t+1}^p \\ 0 \text{ if } R_{PF,t+1} > -VaR_{t+1}^p \end{cases} \tag{14}$$

Based on equation (14), we construct a sequence $\{I_{t+1}\}_{t=1}^{T}$ across T days. Assuming perfect VaR, and the availability of all information at the time of VaR forecast, we should not be able to predict whether the VaR will be exceeded. Forecast of a VaR exceedance should be 100p% every day. Thus the hit sequence of exceedances should be completely unpredictable and thus distributed independently over time as a Bernoulli variable.

3. Data and In Sample Testing

Using the quasi maximum likelihood estimation approach we estimate the jump based GARCH models using FTSE100, NASDAQ100 and S&P100 returns from 1/1/1985 to 31/12/2003 respectively. Excluding weekends we have a total of 4,957 observations for each index. Table 1 provides summary statistics for all three indices studied.

Table 1
Index Summary Statistics

Index	Mean	S. Dev	Skew	Kurt	Min	Max
FTSE100	0.0003	0.0106	-0.7595	10.5831	-0.1303	0.0760
NASDAQ100	0.0005	0.0183	-0.0938	6.9769	-0.1634	0.1720
S&P100	0.0004	0.0115	-1.8903	40.6403	-0.2369	0.0854

The returns are based on daily observations. In Table 1 Skew represents skewness which for a series following the normal distribution should be zero. Kurt represents excess kurtosis which again should be zero for a series following the normal distribution. All three series display significant negative skewness and excess kurtosis. The negative skewness implies that all three series experience significant jumps down but only gradual moves up and the excess kurtosis implies that there are far more extreme observations on a daily basis than would be predicted by the normal distribution. The negative skewness and excess kurtosis is consistent with the findings in current finance literature.

Table 2 - Table 4 provide the estimated parameters for the three indices. The four different combinations of models considered are GARCH(1,1), NGARCH(1,1), GARCH(1,1) with jumps and NGARCH(1,1) with jumps. Turning to Table 2, we find that for the FTSE100, jumps are very rare. However, when jumps do occur they are negative with large mean and variance. For example, the probability of a jump both with GARCH(1,1) specification and NGARCH(1,1) specification is less 0.5% per day.

Table 2
Parameter Estimate for FTSE100

Parameter	GARCH	GARCHJ	NGARCH	NGARCHJ
$\omega \times 10^{-3}$	0.0022	0.0015	0.0000	0.0017
α	0.0844	0.0714	0.0755	0.0637
	(0.0096)	(0.0082)	(0.0046)	(0.0077)
β	0.8959	0.9083	0.8816	0.9012
	(0.0125)	(0.0107)	(0.0073)	(0.0109)
θ			0.5457	0.4910
			(0.0000)	(0.0812)
μ		-0.0156		-0.0128
		(0.0110)		(0.0116)
γ		0.0346		0.0357
		(0.0095)		(0.0105)
q		0.0046		0.0045
		(0.0028)		(0.0029)

Notes: ω, a, β, θ are GARCH parameters. μ is the standard deviation in jumps, γ is the standard deviation in jumps and q is the jump intensity. The numbers in parentheses are standard errors. We note that mean value of the sizes are negative. This is consistent with the findings of other researchers.

Table 3
Parameter Estimate for NASDAQ100

Parameter	GARCH	GARCHJ	NGARCH	NGARCHJ
$\omega \times 10^{-3}$	0.0025	0.0008	0.0036	0.0016
α	0.0796	0.0525	0.0821	0.0615
	(0.0086)	(0.0072)	(0.0090)	(0.0074)
β	0.9133	0.9392	0.8911	0.9129
	(0.0094)	(0.0082)	(0.0112)	(0.0108)
θ			0.4731	0.5121
			(0.0555)	(0.0701)
μ		-0.0090		-0.0084
		(0.0050)		(0.0052)
γ		0.0312		0.0271
		(0.0046)		(0.0046)
q		0.0249		0.0291
		(0.0089)		(0.0133)

Notes: ω, a, β, θ are GARCH parameters. μ is the standard deviation in jumps, γ is the standard deviation in jumps and q is the jump intensity. The numbers in parentheses are standard errors. We note that mean value of the sizes are negative. This is consistent with the findings of other researchers.

Table 4
Parameter Estimate for S&P100

Parameter	GARCH	GARCHJ	NGARCH	NGARCHJ
$\omega \times 10^{-3}$	0.0010	0.0000	0.0000	0.0000
α	0.0772	0.0762	0.0677	0.0497
	(0.0071)	(0.0000)	(0.0018)	(0.0013)
β	0.9163	0.9652	0.8662	0.9003
	(0.0083)	(0.0005)	(0.0015)	(0.0000)
θ			0.9093	0.8557
			(0.0000)	(0.0000)
μ		-0.0124		-0.0133
		(0.0053)		(0.0000)
γ		0.0423		0.0321
		(0.0000)		(0.0000)
q		0.0296		0.0156
		(0.0021)		(0.0000)

Notes: ω, a, β, θ are GARCH parameters. μ is the standard deviation in jumps, γ is the standard deviation in jumps and q is the jump intensity. The numbers in parentheses are standard errors. We note that mean value of the sizes are negative. This is consistent with the findings of other researchers.

From Table 3 we observe the same trends; however, with the NASDAQ100 mean jump sizes are smaller than for FTSE100. The probability of a jump is still very low but significantly greater than the FTSE100.

Table 4 shows that the S&P100 mean jump sizes are fractionally larger than the NASDAQ100 jumps. Table 4 also shows that for the S&P100 index, the probability of a jump in NGARCH setting is almost half as in a GARCH setting. Overall Table 2 - Table 4 demonstrate that jumps are infrequent, they are negative with a large mean. These findings are consistent with the findings of other researchers, for example see Eraker, Johannes and Polson (2003).

For each business day from 1st January 1987 to 31st December 1987 we In-Sample VaR test based on RiskMetrics, Monte Carlo simulation and Filtered Historical simulation over different horizons. For RiskMetrics we set =0.94 and for FHS we use the previous two years data for updating. We perform 5000 simulations. In each of the In-Sample test we store risk measures for four different horizons (1, 2, 5, 10) and four different probability levels (0.90, 0.95, 0.99, 0.995, 0.997, 0.998). We compare the actual loss over the horizon with VaR over the same horizon to note whether any VaR exceedances has occurred. Results of the In-Sample tests are summarised in Table 5 - Table 7. Examination of the tables leads to the following conclusions. RiskMetrics fails as the confidence interval increases. For the FTSE100 and S&P100 GARCH model leads to fewer VaR exceedances than NGARCH model. Augmenting GARCH and NGARCH models with jumps leads to lower number of VaR exceedances at long horizon dates. In some instances the improvement is significant. For example from Table 7, at 90% confidence interval, using Monte Carlo simulation, GARCH violation is 8.39%, compared to 0.88% when GARCH augmented with jumps is used. Generally up to a confidence interval of 99.5%, we find introduction of jumps into the GARCH and NGARCH model lowers the number of exceedances to acceptable levels. At confidence intervals greater than 99.5% and for long horizons all models and methods fail. For example from Table 5 at 99.8% confidence interval with 10 days horizon, exceedances range from 0.38% to 0.46%.

In summary the worst performing approach is the RiskMetrics, followed by the GARCH and GARCH augmented with jumps. Filtered Historical simulation does not lead to fewer violations as would be the case if Filtered Historical simulation was more accurate in VaR calculation than Monte Carlo simulation. This is surprising as we would expect Filtered Historical simulation to be more accurate as it uses more recent data to produce the standardised returns used for simulating next period returns. Most importantly all methods fail at long horizon and high confidence levels. Indeed none of the methods seem to have any advantages over the other. This result is consistent with the findings of Christoffersen et al. (2001). Our results thus indicate that when stock markets crash as for example during 1987, then the widely used parametric approach to *VaR* calculation using RiskMetrics or GARCH with and without jumps fail. Thus our findings indicate that the only way to prepare for stock market crashes of significant scale is to perform stress testing under very extreme conditions to establish the appropriate level of *VaR* and to use these *VaR*'s rather than the *VaR*'s estimated using parametric approaches.

Table 5
In-Sample Testing for FTSE10

CI	1		3		5		10	
90%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	9.39		9.89		9.85		9.97	
GARCH	7.40	9.93	8.55	11.15	8.66	11.31	7.74	11.77
GARCHJ	8.36	9.77	8.93	10.73	8.85	10.66	7.97	10.85
NGARCH	12.57	10.35	13.84	11.23	13.72	11.54	14.41	12.42
NGARCHJ	7.90	9.85	8.70	10.54	8.43	10.50	7.44	10.77
95%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	4.64		5.56		5.71		5.17	
GARCH	3.60	4.79	4.14	5.52	4.14	6.32	3.72	6.36
GARCHJ	3.99	4.60	4.37	5.25	4.41	5.40	3.76	4.91
NGARCH	7.90	4.64	8.32	5.67	8.89	5.94	8.66	5.56
NGARCHJ	3.64	4.52	3.87	4.98	3.83	5.37	2.99	4.52
99%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	1.49		1.65		1.99		1.42	
GARCH	1.11	1.15	0.92	1.23	0.92	1.11	0.65	1.03
GARCHJ	1.11	0.92	0.84	0.65	0.65	0.69	0.46	0.46
NGARCH	2.64	1.30	2.68	1.03	2.84	1.15	2.53	0.69
NGARCHJ	1.15	0.92	0.69	0.65	0.42	0.61	0.42	0.50
99.5%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	1.00		0.96		1.07		0.73	
GARCH	0.77	0.69	0.50	0.57	0.38	0.38	0.42	0.57
GARCHJ	0.73	0.42	0.31	0.31	0.38	0.31	0.38	0.38
NGARCH	2.11	0.65	1.69	0.69	1.76	0.42	1.15	0.46
NGARCHJ	0.65	0.46	0.27	0.35	0.31	0.27	0.38	0.38

99.7%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	0.84		0.77		0.77		0.61	
GARCH	0.65	0.42	0.42	0.46	0.31	0.38	0.61	0.38
GARCHJ	0.38	0.19	0.19	0.19	0.23	0.27	0.42	0.38
NGARCH	1.76	0.42	1.38	0.54	1.11	0.35	0.38	0.38
NGARCHJ	0.42	0.27	0.15	0.15	0.23	0.27	0.38	0.38
99.8%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	0.65		0.73		0.50		0.46	
GARCH	0.46	0.35	0.27	0.38	0.27	0.35	0.42	0.38
GARCHJ	0.27	0.15	0.12	0.12	0.23	0.23	0.38	0.38
NGARCH	1.53	0.35	1.07	0.42	0.77	0.27	0.54	0.38
NGARCHJ	0.38	0.15	0.12	0.12	0.19	0.19	0.38	0.38

Table 6
In-Sample Testing for NASDAQ100

CI	1		3		5		10	
90%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	9.39		9.89		9.85		9.97	
GARCH	5.52	9.97	6.71	11.08	6.63	11.50	5.48	11.69
GARCHJ	7.05	9.31	7.36	9.12	6.82	6.82	8.32	4.94
NGARCH	5.06	9.89	5.40	11.19	5.52	11.00	4.22	11.69
NGARCHJ	6.21	8.82	5.86	8.70	5.44	8.09	3.80	7.40
95%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	4.64		5.56		5.71		5.17	
GARCH	2.49	4.75	3.18	5.48	2.91	6.17	2.38	6.29
GARCHJ	3.30	3.83	3.14	3.79	2.80	3.33	1.84	2.53
NGARCH	2.15	4.60	2.57	5.25	2.30	5.83	1.53	5.44
NGARCHJ	2.57	3.60	2.38	3.37	1.95	3.03	1.11	2.34
99%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	1.49		1.65		1.99		1.42	
GARCH	0.77	1.11	0.50	1.19	0.42	1.19	0.42	1.07
GARCHJ	0.50	0.35	0.19	0.19	0.23	0.27	0.38	0.38
NGARCH	0.61	1.03	0.35	1.07	0.31	1.07	0.38	0.73
NGARCHJ	0.42	0.31	0.19	0.19	0.23	0.23	0.38	0.38
99.5%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	1.00		0.96		1.07		0.73	
GARCH	0.46	0.65	0.35	0.57	0.31	0.73	0.42	0.54
GARCHJ	0.08	0.08	0.12	0.12	0.23	0.19	0.38	0.38
NGARCH	0.42	0.65	0.23	0.61	0.19	0.57	0.38	0.50
NGARCHJ	0.08	0.08	0.12	0.12	0.19	0.19	0.38	0.38
99.7%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	0.84		0.77		0.77		0.61	
GARCH	0.38	0.42	0.31	0.50	0.27	0.38	0.38	0.38
GARCHJ	0.08	0.08	0.12	0.12	0.23	0.19	0.38	0.38
NGARCH	0.31	0.38	0.15	0.46	0.19	0.35	0.38	0.38
NGARCHJ	0.08	0.08	0.12	0.12	0.19	0.19	0.38	0.38
99.8%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	0.65		0.73		0.50		0.46	
GARCH	0.27	0.35	0.19	0.38	0.19	0.38	0.38	0.38
GARCHJ	0.08	0.08	0.12	0.12	0.19	0.19	0.38	0.38
NGARCH	0.12	0.31	0.12	0.38	0.19	0.23	0.38	0.38
NGARCHJ	0.08	0.08	0.12	0.12	0.19	0.19	0.38	0.38

Table 7
In-Sample Testing for S&P100

CI	1		3		5		10	
90%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	9.39		9.89		9.85		9.97	
GARCH	7.97	10.20	8.89	11.12	9.05	11.46	8.39	11.69
GARCHJ	2.22	9.08	2.30	8.43	1.61	7.21	0.88	5.10
NGARCH	12.50	10.27	13.11	10.92	13.07	11.58	13.45	12.07
NGARCHJ	12.27	9.35	12.07	9.24	11.42	9.01	9.89	8.66
95%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	4.64		5.5		5.71		5.17	
GARCH	3.79	4.91	4.64	5.67	4.45	6.48	4.14	6.52
GARCHJ	0.84	3.64	0.57	2.72	0.38	1.99	0.42	1.11
NGARCH	7.51	4.98	7.51	5.25	8.09	5.86	7.59	5.06
NGARCHJ	6.94	4.29	6.21	3.72	5.48	3.68	3.53	2.57
99%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	1.49		1.65		1.99		1.42	
GARCH	1.19	1.07	1.03	1.15	0.96	1.26	0.69	0.92
GARCHJ	0.08	0.15	0.12	0.15	0.23	0.19	0.38	0.38
NGARCH	2.61	1.23	2.22	1.15	2.34	0.96	1.61	0.69
NGARCHJ	1.69	0.46	0.38	0.23	0.31	0.19	0.38	0.38
99.5%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	1.00		0.96		1.07		0.73	
GARCH	0.77	0.69	0.61	0.61	0.42	0.65	0.42	0.57
GARCHJ	0.08	0.08	0.12	0.12	0.19	0.19	0.35	0.38
NGARCH	1.95	0.54	1.72	0.65	1.38	0.42	0.73	0.42
NGARCHJ	0.31	0.15	0.12	0.12	0.23	0.19	0.38	0.38
99.7%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	0.84		0.77		0.77		0.61	
GARCH	0.69	0.42	0.50	0.50	0.35	0.38	0.42	0.38
GARCHJ	0.08	0.08	0.12	0.12	0.19	0.19	0.35	0.38
NGARCH	1.69	0.46	1.07	0.46	1.00	0.31	0.54	0.38
NGARCHJ	0.08	0.08	0.12	0.12	0.19	0.19	0.38	0.38
99.8%	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**	**MC**	**FHS**
RM	0.65		0.73		0.50		0.46	
GARCH	0.54	0.31	0.38	0.38	0.27	0.35	0.42	0.38
GARCHJ	0.08	0.04	0.08	0.12	0.15	0.19	0.31	0.35
NGARCH	1.38	0.31	0.96	0.35	0.61	0.27	0.50	0.38
NGARCHJ	0.08	0.04	0.12	0.12	0.19	0.19	0.38	0.38

4. Conclusions

Value at Risk as an approach plays a central role in the study of market risk. In this study we have In-Sample tested the industry standard RiskMetrics with both standard Monte Carlo simulation and Filtered Historical simulation of Barone-Adesi et-al. Our conclusions are the following. First industry standard RiskMetrics fails at high confidence intervals and long horizon dates. Second the simpler GARCH for certain financial time series may lead to fewer exceedances than the more intuitive and complex NGARCH. When both GARCH and NGARCH models are augmented with jumps, numbers of exceedances were generally reduced even at long horizons. At very high confidence interval levels and long horizons all models fail. This study has demonstrated the inadequacy of standard VaR methodologies during the period 1985 – 2003. In short by implication our study shows that the industry risk measures were not sufficiently robust to meet the challenges to the market posed by the credit crunch. The uses of alternative risk measures such as expected shortfall and spectral risk measure provides an alternative way of assessing the risk in turbulent markets and remains the subject of further studies.

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