

Go Public or not?

Private Firms' IPO Decision and IPO Long-run Underperformance

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We propose a random search-based model to study the searching and matching process between private firms and investment banks in the pre-IPO market. We find out that there exists a close link between private firms' IPO decision and IPO long-run underperformance. Our model shows that the IPO long-run underperformance has already been embedded in the private firm's current decision on going public. Specifically, we introduce the concept of the private firm's "reservation initial offer price". Thus, any realized initial offer price lower than this reservation price only leads to the withdrawal of the IPO by the private firm. This result is also related to the puzzle why a quantity adjustment is more frequent than a price adjustment in an IPO issuing process. Meanwhile, any realized initial offer price higher than this reservation price will ultimately lead to the IPO long-run underperformance. More importantly, our model provides two market equilibrium conditions to pin down the reservation initial offer price for the first time. In addition, we study the factors which may affect the magnitude of the reservation initial offer price R^* and their empirical implications as well.

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1. Introduction

Going public or not is one of the most important decisions which any privately-owned firm needs to make. There are both pros and cons closely associated with this process. For instance, having a firm's stock listed on the stock exchange will significantly enhance the firm's business reputation, thus providing it with broader financing channels. In the meantime, the firm's ownership will be diluted, which none of the founders of the firm can easily confront without some emotional struggles. In addition, the firm is required to disclose its extensive financial and operational information to the market according to the stringent rules of SEC, which is indeed the real cost occurring to the firm.

Once a private firm makes its decision to go public, most of the time, it needs to find an investment bank to underwrite its initial public offering (IPO). When the IPO succeeds, the privately-owned firm will become a public owned company and its shares will then be traded in the secondary market (i.e. the stock exchange) by general investors. During this post-issue secondary market stock transactions there is an intriguing puzzle called IPO long-run underperformance. Ritter (1991) is the first to

find that in the three years following the offering, IPO firms underperformed significantly comparable firms from the same industry and with the similar size. More recently Ritter and Welch (2002) examine 6249 IPOs during 1980-2001 and record that IPO firms have a three-year return, which is 5.1 percent lower than comparable firms. During the period of 1980-2009, if a typical investor bought IPO shares at the first-day closing price and held them for three years, the average IPO would underperform the CRSP value-weighted market index by 19.7 percent and similar companies matched by market capitalization and book-to-market ratio by 7.3 percent.

Ritter (1991) explains this phenomenon with investors being overoptimistic about the future earnings of the IPO firms, and with firms taking advantage of market conditions. Subsequent studies (Brav and Gompers (1997), Brav, Geczy and Gompers (2000), Eckbo, Masulis and Norli (2000) and Eckbo and Norli (2005)) present evidence that long-term IPO underperformance is consistent with the small growth firms exhibiting lower returns (Fama and French (1992)), or the failure of CAPM to explain returns for such firms.

Purnanandam and Swaminathan (2001) find that when initial offer prices are used, IPO firms are priced about 50 percent above comparable non-IPO firms. Although they suggest that this initial overpricing with respect to comparable non-IPO firms could help predict IPO long-run underperformance, they, however, cannot explain why there exists this initial overpricing in the first place. Our search-based model can fill in this gap. In this paper, we construct a random search-based model to study the searching and matching process between private firms and investment banks in the pre-IPO market. We find out that there exists a close link between private firms' IPO decision and IPO long-run post-issue underperformance. Our model indicates that the IPO *long-run* underperformance has already been embedded in the private firm's *current* decision on going public. Specifically, we introduce the concept of the private firm's "reservation initial offer price". Only if the realized initial offer price is larger than the private firm's reservation price will the private firm agree to go public. Therefore, all observed initial offer prices in the IPO market will form a left-truncated distribution when compared to the original distribution consisting of both observed and potential initial offer prices together. Since the mean of the left-truncated distribution is always larger than the mean of the original distribution, the existence of this initial overpricing for IPO firms is thus resolved.

Furthermore, our concept of the private firm's "reservation initial offer price" can also be applied to explain the puzzle why a quantity adjustment is more frequent than a price adjustment in an IPO issuing process. Ritter and Welch (2002) state that: "*Although offer prices are lowered, many firms withdraw their offering rather than proceed with their IPO. In other words, why is there quantity adjustment, rather than price adjustment? This is a puzzle not only for the IPO market, but for follow-on offerings as well.*" According to our model, due to the existence of this reservation initial offer price for the private firm, the initial offer price is downward-inelastic when touching on its

reservation. Any suggested initial offer price which is less than its reservation initial offer price could only lead to no IPO in the first place.

Although the concept of the private firm's "reservation initial offer price" created in this paper belongs to the broad class of "reservation prices/wages" commonly used in auctions and labor markets, we are the first to associate it with IPO long-run underperformance. More importantly, for the first time, our search-based model provides necessary conditions at the market equilibrium to pin down the value of this reservation initial offer price for the private firm, while previous IPO literature only mentions this concept casually and never attempts to model it from the viewpoint of the market microstructure. In another word, our model emphasizes the importance of the market search friction between private firms and investment banks in the pre-IPO market. When the private firm attempts to sell its equity for the first time in the primary security market, it has to search a proper leading investment bank to help underwrite the issue. The fact that the success of IPO depends not only on the effort of the private firm but also on the professional support of the investment bank has a far-reaching effect on the modeling of the private firm's IPO decision under uncertainty. According to our model, the reservation initial offer price will ultimately be determined by the market searching structure summarized in two conditions: the first one is the investment bank's free-entry condition and the second one is the investment bank's share of the profit condition.

2. A pre-IPO search model

In this section, we describe a stylized pre-IPO market including homogeneous private firms (denoted by f) whose final aim is always to go public at a good market timing, and homogeneous investment banks (denoted by b), the support of which is of necessity for the success of an IPO. The initial number of private firms is normalized to 1 and the initial number of investment banks is n . The value of n is usually much smaller than 1 since there are more private firms than investment banks in the pre-IPO market. We assume that during a given time period each private firm can hire only one investment bank to underwrite its IPO and each investment bank can serve only one private firm customer.

Those two types of agents are continuously meeting with each other according to a standard Poisson process with meeting rates of α_f and α_b , respectively. Hence, on average, during each time period each private firm will meet α_f number of investment banks and each investment bank will meet α_b number of private firms. For the same reason, the value of α_b is typically larger than the value of α_f . Moreover, the values of both α_f and α_b cannot be infinite, which characterizes the presence of the search friction existing in the pre-IPO market. The values of α_f and α_b will ultimately depend on the relative number of private firms and investment banks in the market, i.e. the market tightness. The reciprocals of α_f and α_b ($1/\alpha_f$ and $1/\alpha_b$) thus represent the expected meeting time, accounting for not only the time spent on searching, but also the time consumed in the negotiation process by the two agents. Furthermore, the magnitude of these two parameters can even include some type of hindrance

originated from asymmetric information and the heterogeneity of private firms and investment banks.

When meeting with each other, the private firm and the investment bank simultaneously decide whether to form a strategic pair or not. If either agent doesn't agree to form a pair, there will be no IPO later. Some reasons for a private firm to decline to form this pair include: the private firm waits for another better offer from another investment bank or the private firm waits for another good timing to go public. The same logic applies to the consideration of the investment bank. If the private firm and the investment bank both agree to form a strategic pair, the investment bank will require a profit of k , representing any service fees related to underwriting activities such as the commission fee and other un-named benefits, and the private firm will keep the residual part $(R-k)$, here R denotes the total gross proceeds from an IPO. Both k and $(R-k)$ are due when the IPO succeeds in the future. *In our model, we assume that each firm only issues one share of stock. Thus, we ignore the problem of how many shares will be outstanding for an IPO. In this way the total gross proceeds of an IPO can be considered as the initial offer price of an IPO as well.*

We further assume that the total gross proceeds or the initial offer price (R) supported by the current financial market condition is a random variable with the cumulative distribution Function (CDF) of $F(R)$ which is a common knowledge for both agents at the beginning of this game. $F(R)$ has a support of $[0, \bar{R}]$. Here \bar{R} is the maximum possible value of R . However, R will be revealed once the private firm and the investment banks forms a strategic pair. Although private firms and investment banks are *ex ante* homogeneous, different meetings can lead to different values of R . The complication of our search-based models largely depends on the assumption of the probability distribution for R . For one extreme case when the distribution for R degenerates into a point, our model can be significantly simplified since there will be no uncertainty around the IPO. Further restrictions on the feasible value of R can also be imposed from the outside of the market. For instance, any realized value of R lower than a special value would in the first place discourage the formation of a pre-IPO strategic pair between a private firm and an investment bank. In reality, this restraint may reflect the legally minimum initial capital requirement to be listed in a stock exchange.

The investment bank's profit k is the result of bargaining between the private firm and the investment bank when meeting with each other. We utilize the generalized Nash bargaining scheme to pin down the value of k , assuming that the investment bank's bargaining power is characterized by θ . The value of θ falls between 0 and 1. θ approaching 1 indicates that the investment bank has a higher bargaining power over the private firm. Therefore, it can claim a larger amount of the profit from any fixed amount of the total gross proceeds of an IPO and vice versa.

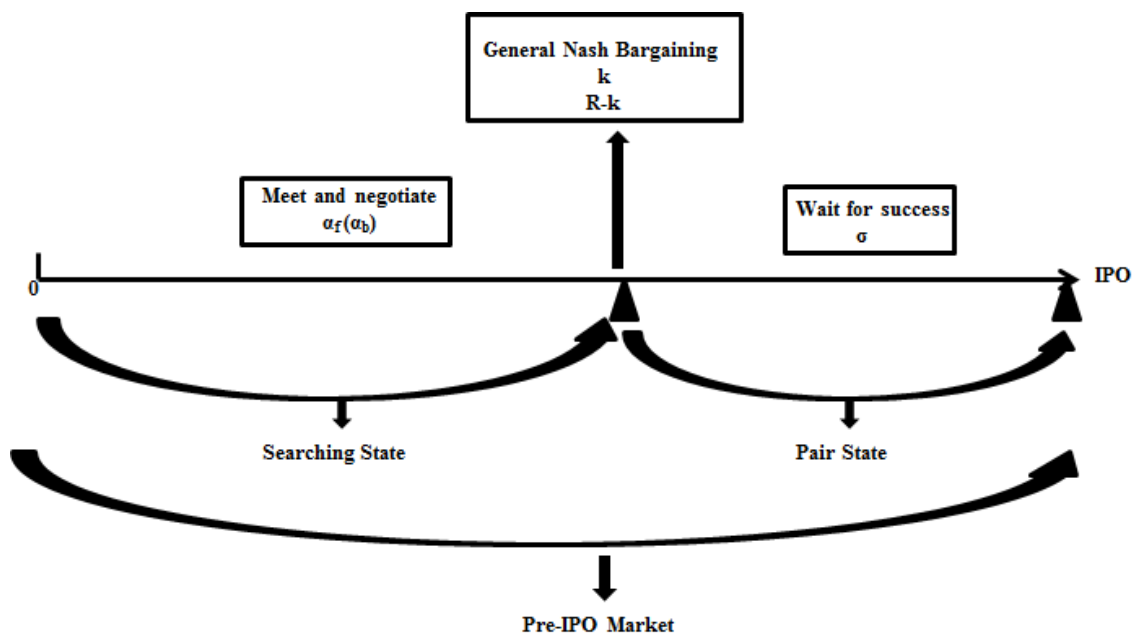
We assume that the occurrence of successful IPOs follows another standard Poisson process with a success arrival rate of σ , i.e. on average during each time

period there are σ number of successful IPOs among all proposed IPOs. The value of σ cannot be infinite either, which implies the concern that any IPO promoted by a strategic pair formed by a private firm and an investment bank is not guaranteed to be successful in the real world. Once an IPO succeeds, the investment bank will return to the market and the private firm will exit the market. Moreover, a clone of the private firm will refill the market to keep the market equilibrium in the language of search theory.

In addition, we assume that both types of agents are risk neutral and the market on-going (risk-free) discount rate is denoted as r , which characterizes the time preference of private firms and investment banks.

In sum, the entire pre-IPO process can be illustrated by Figure I. In the pre-IPO market, private firms and investment banks can stay in two distinguished states: the searching state where private firms and investment banks meet and negotiate with each other and the pair state where the strategic pair formed by one private firm and one investment bank waits for the success of the IPO.

Figure I. The schematic of the pre-IPO market



Since there are two types of agents (b denotes the investment bank and f denotes the private firm) and two states (0 indicates the searching state and 1 indicates the pair state), we thus define four state value functions:

V_f^0 : the value of a private firm who is searching an investment bank in the market;

V_f^1 : the value of a private firm who forms a strategic pair with an investment bank;

V_b^0 : the value of an investment bank who is searching a private firm in the market;

V_b^1 : the value of an investment bank who forms a strategic pair with a private bank.

These four value functions represent corresponding “utilities” or “welfares” obtained when staying in those two states for those two types of agents, respectively.

3. Mathematical model and discussion

3.1 Model set-up

In this section, we apply the basic Bellman search equations to analyze the pre-IPO process between private firms and investment banks. We will define the private firm's reservation initial offer price at the market equilibrium and discuss its empirical implications.

Since there are two types of agents, private firms and investment banks who are continuously searching in the pre-IPO market, the interaction between them is modeled as a two-sided search in contrast to a one-sided search where only one type of agents is actively searching in the market. Here $V_f^1(R)$ and $V_b^1(R)$ are functions of R ; \tilde{R} is a dummy variable for integration. Thus, if the market prevalent value of the investment bank's share of profit is k^* , the four value functions defined in Section 2 satisfy the below four search equations:

$$r V_f^0 = \alpha_f \int_0^{\tilde{R}} \max\{V_f^1(\tilde{R}) - V_f^0, 0\} dF(\tilde{R}) \quad (1)$$

$$r V_f^1(R) = \sigma [R - k^* - V_f^1(R)] \quad (2)$$

$$r V_b^0 = \alpha_b \int_0^{\tilde{R}} \max\{V_b^1(\tilde{R}) - V_b^0, 0\} dF(\tilde{R}) \quad (3)$$

$$r V_b^1(R) = \sigma (k^* + V_b^0 - V_b^1(R)) \quad (4)$$

All four equations have the similar structure: the left hand side is called the flow value, which is always the product of the discount rate and the value for each specific state; the right hand side is the expected value change from the agent's current state, which equates the product of the state-jump rate (such as α_f , α_b and σ) and the value difference between the agent's current state and its next state.

For instance, for Equation (1), the left hand side represents the flow value for a private firm who is searching an investment bank in the market; the right hand side is the private firm's expected value change jumping from the searching state to the pair state *only if* the firm's value staying in the pair state is higher than that staying in the searching state.

When we compare the private firm's value functions in the two states, V_f^0 and $V_f^1(R)$, we can predict the private firm's decision on going public since the private firm always prefers to stay in the state with the higher “utility” or “welfare”. The private firm's decision rule is that if $V_f^1(R)$ is larger than V_f^0 , i.e. the value of the private firm forming a strategic pair with the investment bank is larger than the value of its staying in the searching state, the private firm will go public; otherwise it will not. Without solving the above equation system, we can infer that $V_f^1(R)$ is a non-decreasing function of R , which means that the higher the realized initial offer price, the more “utility” the private firm can acquire from staying in the pair state. More importantly, there should exist a reservation initial offer price R^* at which

$$V_f^1(R^*) = V_f^0 \quad (5)$$

Due to the above property of the private firm's value functions, its value-based decision rule can be transformed to the price comparison. If the realized initial offer price is smaller than the reservation initial offer price, then the private firm's value in the pair state will be less than its value in the searching state (i.e. $R < R^* \rightarrow V_f^1(R) < V_f^1(R^*) = V_f^0$, here the sign " $<$ " comes from the non-decreasing property of $V_f^1(R)$ and the sign " $=$ " comes from the definition of the reservation initial offer price). Thus, it is wise for the private firm to wait a little longer time for the arrival of a better deal. Proposition 1 summarizes the private firm's decision on going public thereafter.

Proposition 1: *If R is a random variable with a CDF of $F(R)$ (the support of R is $[0, \bar{R}]$), there exists a reservation initial offer price R^* for the private firm whose optimal decision rule is that when the realized value of R is less than R^* , the private firm will stay private and when the realized value of R is larger than R^* , it will go public.*

Although the meaning of Proposition 1 is straightforward, its financial implication cannot be over emphasized since Proposition 1 shed light on the puzzle why a quantity adjustment is more frequent than a price adjustment for the IPO market. Ritter and Welch (2002) state: "Although offer prices are lowered, many firms withdraw their offering rather than proceed with their IPO. In other words, why is there quantity adjustment, rather than price adjustment? This is a puzzle not only for the IPO market, but for follow-on offerings as well." Our answer to this puzzle is that: due to the existence of the private firm's reservation initial offer price R^* , the initial offer price R is downward-inelastic when touching on R^* ; thus, any suggested value of the initial offer price which is lower than the reservation initial offer price can only lead to no IPO in the first place. One caveat needs to be addressed here. The "no IPO" prediction of Proposition 1 under the condition that the initial offer price is lower than the reservation initial offer price is firm. However, Proposition 1 only indicates the willingness of the private firm to go public when the opposite is true. In another word, the initial offer price higher than the private firm's reservation initial offer price is only a necessary condition for a successful IPO since it still needs the collaboration from the investment bank and the demand support from the general investors.

Proposition 2 summarizes the relation between the mean of the left-truncated distribution and the mean of the original distribution for the initial offer price, which indicates that the existence of the reservation initial offer price R^* can be the driving force behind IPO long-run underperformance.

Proposition 2: *If R is a random variable with a CDF of $F(R)$ (the support of R is $[0, \bar{R}]$), according to the property of probability distribution, the mean of the new distribution left-truncated by R^* is always larger than the mean of the original distribution, i.e.*

$$E(R | R > R^*)$$

$$\equiv \frac{\int_{R^*}^{\bar{R}} R dF(R)}{1 - F(R^*)} > E(R) \equiv \int_0^{\bar{R}} R dF(R).$$

Purnanandam and Swaminathan (2001) find that when initial offer prices are used, IPO firms are priced about 50 percent above comparables. They suggest that this

initial overpricing with respect to comparables can help predict IPO long-run underperformance. According to Proposition 2, we can interpret the mean of the distribution left-truncated by the reservation initial offer price R^* , $E(R | R > R^*)$, as the average initial offer price only for private firms who go public. We can further interpret the mean of the original distribution, $E(R)$, as the average initial offer price for all private firms, either going public or not. Since the mean of the left-truncated distribution is always larger than the mean of the original distribution, i.e. $E(R | R > R^*) > E(R)$, the initial overpricing of IPO firms with respect to comparables is rather obvious. Furthermore, in some sense $E(R)$ can also represent the intrinsic value of IPO firms in a stock market without growth. As long as we believe that in the long-run IPO firms' market value will revert from the average initial offer price $E(R | R > R^*)$ to their intrinsic value $E(R)$, the main part of IPO underperformance can thus be resolved.

3.2 Solving for the private firm's reservation initial offer price

To solve for the private firm's reservation initial offer price R^* , we first define two surplus functions, $S_f(R)$ and $S_b(R)$, for private firms and investment banks separately as below. Those functions will be used in the private firm and investment bank's bargaining process to help us form an objective function under the framework of generalized Nash bargaining scheme.

$$S_f(R) = V_f^1(R) - V_f^0 \quad (6)$$

$$S_b(R) = V_b^1(R) - V_b^0 \quad (7)$$

Given the market prevalent value of the investment bank's share of profit k^* and the investment bank's bargaining power θ , we apply the generalized Nash bargaining scheme to divide the initial offer price (or the total gross proceeds) R between the private firm and the investment bank:

$$\text{Max } S_f^{1-\theta} S_b^\theta \quad \text{by choosing } k. \quad (8)$$

The market equilibrium requires that the market prevalent value of k^* be consistent with each investment bank's share of profit k resulted from the general Nash bargaining scheme:

$$k = k^* \quad (9)$$

Recall that initially the number of private firms is normalized to 1 and the number of investment banks is n . Let the number of the strategic pairs be m at the steady state, thus the number of un-paired private firms is $1-m$ and the number of un-paired investment banks is $n-m$ at the steady state then. The balanced steady state flow condition requires that:

$$(1-m) \alpha_f = \theta m = (n-m) \alpha_b \quad (10)$$

Define the market tightness (MT) as the ratio of the number of investment banks $(n-m)$ to the number of private firms $(1-m)$ at the steady state:

$$MT = \frac{n-m}{1-m} \quad (11)$$

The matching technology between private firms and investment banks is abstracted in a matching function denoted as π that depends on the numbers of both types of agents in the pre-IPO market. Assuming that π has a constant rate of return

with respect to those two numbers and has a functional form in Equation (12) (δ is a parameter in the function), the meeting rates of α_f and α_b can thus be expressed as a function of the market tightness MT by Equation (13) and (14):

$$\pi = \pi(1-m, n-m) = (1-m)^{1-\delta}(n-m)^\delta \quad (12)$$

$$\alpha_f = \pi/(1-m) = (1-m)^{-\delta}(n-m)^\delta = MT^\delta \quad (13)$$

$$\alpha_b = \pi/(n-m) = (1-m)^{1-\delta}(n-m)^{\delta-1} = MT^{\delta-1} \quad (14)$$

To close up our model, we assume the free entry for investment banks to the underwriting industry, which requires that the value of investment banks searching in the pre-IPO market be equal to a fixed value of L which is the value of them staying out of this market:

$$V_b^o = L \quad (15)$$

When applying the general Nash bargain scheme (Equation (8)), the balanced steady state flow condition (Equation (10)), the investment bank's value of staying out of the underwriting industry (Equation (15)) and the market equilibrium condition (Equation (9)) to the basic search equations (Equation (1-4)), we can reduce the entire system into two conditions in Propositions 3.

Proposition 3: *if R is a random variable with a CDF of F(R) (the support of R is $[0, \bar{R}]$), the entire system is characterized by two equations (the investment bank's profit condition and the investment bank's free entry condition) with two key variables (the reservation initial offer price R^* and the market tightness MT).*

$$R^* = \left[\frac{(1-\theta)MT^\delta}{r} + \frac{\theta MT^{\delta-1}}{r+\sigma} \right] \int_{R^*}^{\bar{R}} [1 - F(R)] dR \quad (16)$$

$$\theta \sigma R^* = [\theta r + (1-\theta)(r+\sigma)MT]L \quad (17)$$

We will use Equation (16) and (17) to pin down the reservation initial offer price R^* with calibrated parameters in section 4. As the first attempt, given that the market tightness MT is fixed, based on Equation (16) only we can find the effects of r and σ on R^* via comparative statics, which is summarized in Corollary 1:

Corollary 1: *If R is a random variable with a CDF of F(R) (the support of R is $[0, \bar{R}]$), the lower the discount rate r and the success arrival rate of IPO σ , the higher the reservation initial offer price R^* for a given market tightness MT, which finally leads to the more severe IPO long-run underperformance.*

In addition, when R^* is solved, the corresponding expected length of each agent staying in the searching state can be expressed as $\frac{1}{\alpha_f[1-F(R^*)]}$ for private firms and $\frac{1}{\alpha_b[1-F(R^*)]}$ for investment banks because the agents are willing to form a strategic pair only if the realized initial offer price R is larger than the reservation initial offer price R^* , which shows in Corollary 2:

Corollary 2: *If R is a random variable with a CDF of F(R) (the support of R is $[0, \bar{R}]$), the expected lengths of private firms and investment banks staying in the searching state are $\frac{1}{\alpha_f[1-F(R^*)]}$ and $\frac{1}{\alpha_b[1-F(R^*)]}$ respectively.*

4. Empirical implications

In this section, we first calibrate the key parameters of our model according to the typical data from the IPO market. Then we combine the theoretical predictions of our model with the simulation results to illustrate the empirical implications of our model.

4.1 Parameter calibration

We use the median number of IPOs per month from 1980 to 2011 as the success arrival rate of IPO. So, we choose 15 times per month for σ . The reason why we don't use the mean is because the median excludes the extreme effects of the stock market crisis such as 1998-1999 and 2008-2009. Without additional information, we always assume that the matching parameter δ and the investment bank's bargaining power θ are 0.5. While we can use the current risk-free interest rate as the discount rate r in our model, since the current risk-free interest rate is almost zero, we choose the median of monthly 10-year Treasury constant maturities nominal yields from January 1980 to December 2011, which is approximately 0.5% per month, as the discount rate applied in our model. Furthermore, we use the average market value of investment banks, roughly \$20 billion, as the estimated value of investment banks staying out of the market.

Table 1 summarizes the key parameters and their typical values used in our model simulation.

Table 1: Parameters and Their Values Used in the Model

Parameter	Notation	Typical Value
The success arrival rate of IPO	σ	15/month
The parameter in the matching technology function π	δ	0.5
The (risk-free) discount rate	r	0.5%/month
The investment bank's bargaining power	θ	0.5
The value of investment banks staying out of the market	L	\$20 billion

4.2 Simulation results

(A) The reservation initial offer price and IPO long-run underperformance

Besides the typical values in Table 1, we need to assume a functional form and a support for the cumulative distribution function of $F(R)$. Let's use the simplest one for $F(R)$:

$$F(R) = R/0.3, \quad \text{here } R \in [0, 0.3]$$

From 2000 to 2011 there are 1519 offerings with \$352.616 billion of gross proceeds in total. So, the average gross proceeds of a successful IPO transaction is around \$0.2billion. Thus, we choose \$0.3billion as the upper limit of R . This means that the initial offer price is uniformly distributed on the interval of $[0, 0.3]$, i.e. it is equally possible for R to be any value of $[0, 0.3]$. To be noted that the assumption on

the functional form of the probability distribution of R could have a great impact on the simulation results.

One of the key contributions of our random version of search model is to introduce the concept of the reservation initial offer price R^* under the framework of market searching. Furthermore, our model provides a unique and tractable structure to quantify this value with as few assumptions as possible. More importantly, our model links the reservation initial offer price R^* with the puzzle of IPO long-run underperformance. Therefore, the first task in this part is to illustrate how to determine the reservation initial offer price R^* .

Figure II shows the general equilibrium solution for the reservation initial offer price R^* and the market tightness MT at the point where the dark black line (Equation (16)) comes across the light black line (Equation (17)). Here the dark black line denotes the investment bank's profit condition and the light black line represents the investment bank's free entry condition. The reservation initial offer price at the market equilibrium R^* has to satisfy both conditions simultaneously. The reservation initial offer price R^* is \$0.192 billion and the corresponding market tightness MT is 0.009246. Don't confuse this equilibrium reservation initial offer price R^* resulted from our model with the average gross proceeds of a successful IPO transaction (or the initial offer price R) used previously. The reservation initial offer price R^* is an unobservable variable, embedded in private firms' mind and implicitly determined by the model structure while the average of the initial offer price R is observable from the market data.

Once we know the value of R^* , the mean of the truncated distribution can be calculated as $E(R | R > 0.192) = \frac{\int_{0.192}^{0.3} R dF(R)}{1 - F(0.192)} = 0.246$, which is about 64 percent ($\frac{0.246 - 0.15}{0.15} = 64\%$) higher than the unconditional mean $E(R) = \int_0^{0.3} R dF(R) = 0.15$. In terms of IPO language, when initial offer prices are used, on average IPO firms are priced about 64 percent above all the private firms in the pre-IPO market. This simulation result is well consistent with the empirical observations and is considered as the key driving force for IPO long-run underperformance.

According to Equation (13) and (14), with the equilibrium value of the market tightness MT we can easily derive the two meeting rates of α_f and α_b . Here $\alpha_f = MT \delta = 0.009246^{0.5} \approx 0.1$ per month and $\alpha_b = MT^{\delta-1} \approx 10$ per month. This result confirms the estimated values for those two meeting rates from Chen, Petrova and Song (2015).

Next step, we investigate the factors which may affect the magnitude of the reservation initial offer price R^* . To simplify our calculation, we assume that the market tightness MT is fixed and the two corresponding meeting rates of α_f and α_b are 0.1 per month and 10 per month, respectively. Then we can use Equation (16) to solve for R^* and study the effects of the discount rate (r) and the success arrival rate of IPO (σ) on the reservation initial offer price R^* and their implications on IPO long-run underperformance. This procedure is considered as the partial equilibrium analysis for R^* since we don't combine Equation (16) and (17) together.

Figure II. Determination of R^* and MT at the market equilibrium

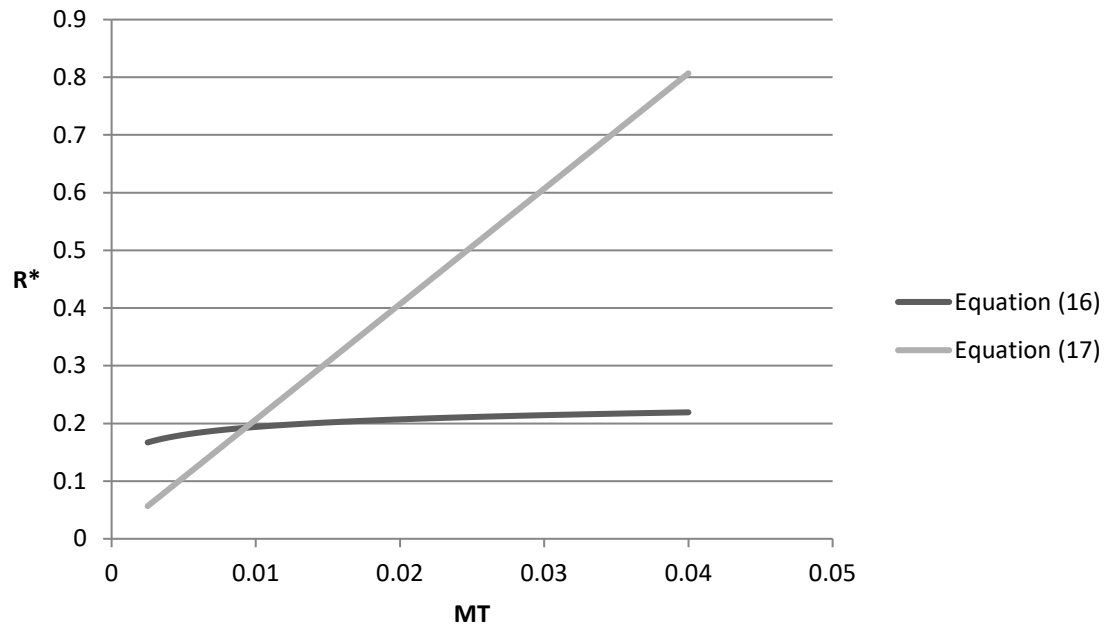
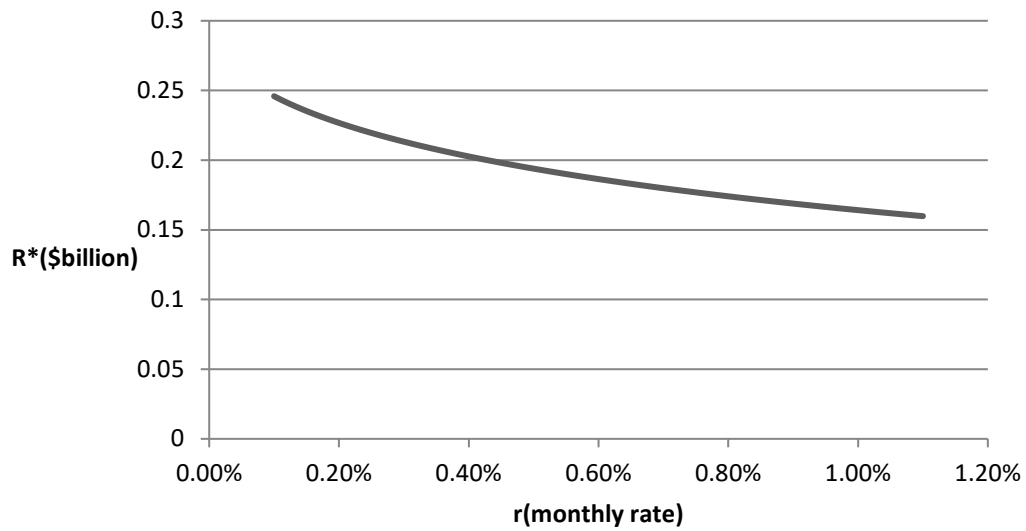


Figure III. The impact of r on R^* when MT is fixed



(B) The impact of the discount rate r on R^* and its empirical implication

In Figure III, when the monthly discount rate increases from 0.1% to 1.1% (i.e. the annual rate falls in between 1.2% to 13.2%), the reservation initial offer price R^* decreases from \$0.246 billion to \$0.160 billion correspondingly.

Table 2 summarizes the impact of the discount rate on the reservation initial offer price and its impact on the overvaluation of the IPO firms. We find that when the central bank's monetary policy is expansionary (i.e. the discount rate r is relative lower), the private firm's reservation initial offer price is higher. Thus, the overvaluation of IPO firms over all the private firms becomes more intense, which will finally lead to a more severe IPO long-run underperformance.

Table 2: The impact of the discount rate

r	R^* (\$Billion)	$E(R R > R^*)$ (\$billion)	Overvaluation over Comparables
0.1%	0.246	0.273	82%
0.5%	0.194	0.247	65%
1.1%	0.160	0.230	53%

(C) The impact of the success arrival rate of IPO σ on R^* and its empirical implication

In Figure IV, when the success arrival rate of IPO σ increases from 1 to 30 per month, the reservation initial offer price decreases from \$0.209 billion to 0.193 billion correspondingly. While there exists a negative relationship between the success arrival rate of IPO and the reservation initial offer price, the entire range of σ can be divided into two phases: [1, 10] and [10, 30]. When σ falls into [1, 10], the curve has a larger negative slope than that when σ belongs to [10, 30]. This phenomenon mainly comes from the relative magnitude of two rates, σ and α_b . If σ is less than α_b (which has an estimated value of 10 per month in our model), then the controlling step will be the strategic pair state and the success arrival rate of IPO will dominate the entire IPO process. Thus, the change of σ will impose a stronger influence on the private firm's reservation initial offer price R^* . Meanwhile, if σ is larger than α_b , the meeting rates in the searching state will become a binding condition. Thus, any change of σ will have a less effect on the private firm's reservation initial offer price R^* .

Moreover, a smaller value of σ means that the IPO market is harder, which also indicates that the role played by the investment bank for an IPO transaction is more important. This is equivalent to a larger bargaining power for the investment bank. Thus, for any fixed size of the pie from an IPO, the investment bank would acquire more share of profit and the private firm would obtain less. Confronting the situation, the private firm will increase its reservation initial offer price (R^*) in the first place in order to let the size of each IPO pie become larger. Therefore, we have a negative relationship between the success arrival rate of IPO (σ) and the reservation initial offer price (R^*).

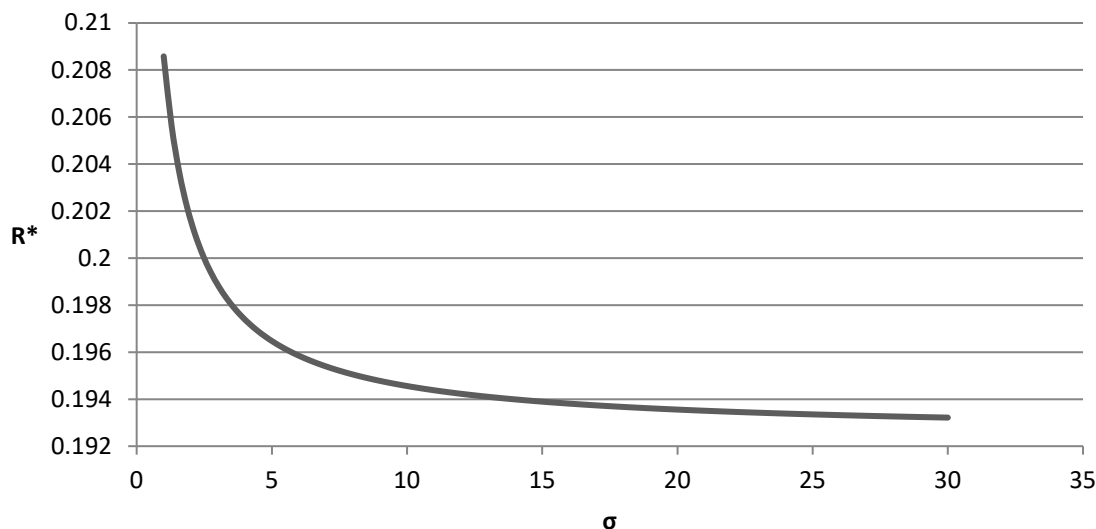
Figure IV. The impact of σ on R^* when MT is fixed

Table 3 summarizes the impact of the success arrival rate of IPO on the reservation initial offer price and its further impact on the overvaluation of the IPO firms. We find that when the success arrival rate of IPO is lower, the private firm's reservation initial offer price is higher. Then the overvaluation of IPO firms becomes more profound, which will have the same effect on IPO long-run underperformance as the discount rate does.

Table 3: The impact of the success arrival rate of IPO

σ	R^* (\$Billion)	$E(R R > R^*)$ (\$billion)	Overvaluation over Comparables
1	0.209	0.2545	70%
15	0.194	0.2470	65%
30	0.193	0.2465	64%

5. Conclusion

In this paper, the optimal strategies of private firms who are eager to go public and investment banks that are assumed to be necessary to serve the IPO process are simultaneously investigated under the framework of two-sided search theory. Four useful value functions for both types of agents are established to represent the corresponding utilities obtained when staying in two distinct states, the searching state and the pair state. One important characteristic of our model is that the intent of every private firm who always wants to go to public is compared with the revealed result that only some of private firms with the realized initial offer price is higher than its reservation initial offer price will go public successfully.

Aided by this model, the complex IPO process can be reduced into a system with a finite number of equations and a finite number of variables, making the research exploration in IPO areas more tractable. Our model explains why there exists an initial overvaluation for IPO firms with respect to comparable non-IPO firms, which

is tightly related to long-run post-issue underperformance. Thus, IPO long-run underperformance can be creatively explained from the "search" angle. Our model suggests that the existence of the private firm's reservation initial offer price be the main driving force behind IPO long-run underperformance, which also explains the puzzle why a quantity adjustment is more frequent than a price adjustment in an IPO issuing process.

Besides the introduction of the concept of the private firm's "reservation initial offer price", empirically, we are pioneers to pin down this value numerically under the framework of market searching. We further find that a lower value of the discount rate and the success arrival rate of IPO can both lead to a higher reservation initial offer price for a given market tightness MT , which finally causes a more severe IPO long-run underperformance.

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Appendix A: Notations

f	private firm
b	investment bank
1	the initial number of private firms normalized to 1
n	the initial number of investment banks
m	the equilibrium number of IPO pairs
α_f	the meeting rate of a private firm to an investment bank, i.e. how many investment banks a private firm can meet during each time period.
α_b	the meeting rate of an investment to a private firm, i.e. how many private firms an investment bank can meet during each time period.
σ	the success arrival rate of IPO
R	the total proceeds of an IPO or the initial offer price of an IPO
k	investment bank's share of profit from an IPO
k^*	investment bank's share of profit from an IPO at the market equilibrium
State "0"	the searching state
State "1"	the IPO pair state
V_f^0	the value of a private firm who is searching an investment bank in the market
V_f^1	the value of a private firm who forms a strategic pair with an investment bank
V_b^0	the value of an investment bank who is searching a private firm in the market
V_b^1	the value of an investment bank who forms a strategic pair with a private bank
r	the (risk-free) discount rate
S_f	the surplus function for a private firm, equals $V_f^1 - V_f^0$
S_b	the surplus function for a private firm, equals $V_b^1 - V_b^0$
θ	investment bank's bargaining power, the parameter in the Nash bargaining scheme
MT	the market tightness, equals $\frac{n-m}{1-m}$, denotes the ratio of the number of investment banks to the number of private firms at the market equilibrium
π	the matching technology function between f and b
δ	the parameter in the matching technology function π
L	the value of investment banks staying out of the market
R^*	the reservation initial offer price of an IPO
F(R)	the cumulative distribution function (CDF) of the initial offer price of R

Appendix B: Proofs of Propositions and Corollaries

Proposition 1

$V_f^1(R)$ and $V_b^1(R)$ are both non-decreasing functions of R . Suppose there exists a reservation initial offer price R^* at which

$$V_f^1(R^*) = V_f^0 \quad (\text{B-1})$$

If $R < R^*$, then $V_f^1(R) < V_f^0$ because $V_f^1(R) < V_f^1(R^*) = V_f^0$, the sign “<” comes from the non-decreasing property of $V_f^1(R)$ and the sign “=” comes from the definition of the reservation initial offer price (B-1).

Proposition 2

According to the property of any probability distribution, the mean of the left-truncated distribution is always larger than the unconditional mean of the original distribution. If we apply this condition to the distribution for the initial offer price, we can reach the conclusion of Proposition 2.

Proposition 3

Before deriving Equation (16) and (17), let's first derive several useful equations. The total surplus S of an IPO is a function of R , i.e. $S = S(R) = S_b(R) + S_f(R)$.

$$S(R) = \frac{\sigma k - r V_b^0}{r + \sigma} + \frac{\sigma(R - k) - (r + \sigma)V_f^0}{r + \sigma} = \frac{\sigma R - (r + \sigma)V_f^0 - r V_b^0}{r + \sigma} \quad (\text{B-2})$$

In the above equation, the division of R between f and b is unrelated to the total surplus S , i.e. k does not show in (B-2). When $R = R^*$, according to the definition of R^* from (B-1), $S(R^*) = 0$. So, the numerator of (B-2) equals to 0 at $R = R^*$, i.e.

$$\sigma R^* = (r + \sigma)V_f^0 + r V_b^0 \quad (\text{B-3})$$

In addition, according to the F.O.C. for the generalized Nash bargain problem:

$$\sigma k^* = \theta \sigma R + (1 - \theta)r V_b^0 - \theta(r + \sigma)V_f^0 \quad (\text{B-4})$$

Now we use (B-3) and (B-4) to derive (16):

$$(2) \rightarrow V_f^1(R) = \frac{\sigma(R - k^*)}{r + \sigma} \quad (\text{B-5})$$

Insert (B-5) into (1) and re-format (1) according to Proposition 1:

$$\begin{aligned} r V_f^0 &= \alpha_f \int_0^{\bar{R}} \max\{V_f^1(\bar{R}) - V_f^0, 0\} dF(\bar{R}) = \alpha_f \int_{R^*}^{\bar{R}} \left[\frac{\sigma(R - k^*)}{r + \sigma} - V_f^0 \right] dF(R) \\ &= \frac{\alpha_f}{r + \sigma} \int_{R^*}^{\bar{R}} [\sigma(R - k^*) - (r + \sigma)V_f^0] dF(R) \end{aligned} \quad (\text{B-6})$$

Insert (B-4) into (B-6):

$$r V_f^0 = \frac{\alpha_f}{r + \sigma} \int_{R^*}^{\bar{R}} [\sigma R - \theta \sigma R - (1 - \theta)r V_b^0 + \theta(r + \sigma)V_f^0 - (r + \sigma)V_f^0] dF(R) \quad (\text{B-7})$$

Simplify (B-7) via (B-3):

$$\begin{aligned} r V_f^0 &= \frac{\alpha_f}{r + \sigma} \int_{R^*}^{\bar{R}} [(1 - \theta)\sigma R - (1 - \theta)r V_b^0 - (1 - \theta)(r + \sigma)V_f^0] dF(R) \\ &= \frac{\alpha_f}{r + \sigma} \int_{R^*}^{\bar{R}} \{(1 - \theta)\sigma R - (1 - \theta)[r V_b^0 + (r + \sigma)V_f^0]\} dF(R) \\ &= \frac{\alpha_f}{r + \sigma} \int_{R^*}^{\bar{R}} [(1 - \theta)\sigma R - (1 - \theta)\sigma R^*] dF(R) \\ &= \frac{\alpha_f(1 - \theta)\sigma}{r + \sigma} \int_{R^*}^{\bar{R}} (R - R^*) dF(R) \end{aligned} \quad (\text{B-8})$$

Integrate (B-8) by parts:

$$rV_f^0 = \frac{\alpha_f(1-\theta)\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \quad (B-9)$$

In the same way, we can derive a formula for rV_b^0 :

$$(4) \rightarrow V_b^1(R) = \frac{\sigma(k^* + V_b^0)}{r+\sigma} \quad (B-10)$$

Insert (B-10) into (3) and re-format (3) according to Proposition1:

$$\begin{aligned} rV_b^0 &= \alpha_b \int_0^{\bar{R}} \max\{V_b^1(\bar{R}) - V_b^0, 0\} dF(\bar{R}) = \alpha_b \int_{R^*}^{\bar{R}} \left[\frac{\sigma(k^* + V_b^0)}{r+\sigma} - V_b^0 \right] dF(R) \\ &= \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\sigma(k^* + V_b^0) - (r + \sigma)V_b^0] dF(R) \\ &= \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\sigma k^* - rV_b^0] dF(R) \end{aligned} \quad (B-11)$$

Insert (B-4) into (B-11):

$$rV_b^0 = \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\theta\sigma R + (1 - \theta)rV_b^0 - \theta(r + \sigma)V_f^0 - rV_b^0] dF(R) \quad (B-12)$$

Simplify (B-12) via (B-3):

$$\begin{aligned} rV_b^0 &= \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\theta\sigma R + (1 - \theta)rV_b^0 - \theta(r + \sigma)V_f^0 - rV_b^0] dF(R) \\ &= \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\theta\sigma R - \theta(r + \sigma)V_f^0 - \theta rV_b^0] dF(R) \\ &= \frac{\alpha_b}{r+\sigma} \int_{R^*}^{\bar{R}} [\theta\sigma R - \theta\sigma R^*] dF(R) \\ &= \frac{\alpha_b\theta\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} (R - R^*) dF(R) \end{aligned} \quad (B-13)$$

Integrate (B-13) by parts:

$$rV_b^0 = \frac{\alpha_b\theta\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \quad (B-14)$$

Multiply (B-9) by $\frac{r+\sigma}{r}$:

$$\begin{aligned} rV_f^0 \left(\frac{r+\sigma}{r} \right) &= \left(\frac{r+\sigma}{r} \right) \frac{\alpha_f(1-\theta)\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \\ (r + \sigma)V_f^0 &= \frac{\alpha_f(1-\theta)\sigma}{r} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \end{aligned} \quad (B-15)$$

Add (B-14) and (B-15) together, we get:

$$(r + \sigma)V_f^0 + rV_b^0 = \frac{\alpha_f(1-\theta)\sigma}{r} \int_{R^*}^{\bar{R}} [1 - F(R)] dR + \frac{\alpha_b\theta\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \quad (B-16)$$

Recall (B-3), the left-hand side equals σR^* , then,

$$\begin{aligned} \sigma R^* &= \frac{\alpha_f(1-\theta)\sigma}{r} \int_{R^*}^{\bar{R}} [1 - F(R)] dR + \frac{\alpha_b\theta\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \\ R^* &= \left[\frac{\alpha_f(1-\theta)}{r} + \frac{\alpha_b\theta}{r+\sigma} \right] \int_{R^*}^{\bar{R}} [1 - F(R)] dR \end{aligned} \quad (B-17)$$

Replace α_f by MT^δ and α_b by $MT^{\delta-1}$ in (B-17), we get (16):

$$R^* = \left[\frac{(1-\theta)MT^\delta}{r} + \frac{\theta MT^{\delta-1}}{r+\sigma} \right] \int_{R^*}^{\bar{R}} [1 - F(R)] dR \quad (16)$$

Now we use (15), (16) and (B-14) to derive (17):

$$\begin{aligned} rV_b^0 = rL &= \frac{\alpha_b\theta\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR = \frac{MT^{\delta-1}\theta\sigma}{r+\sigma} \int_{R^*}^{\bar{R}} [1 - F(R)] dR \\ &= \frac{MT^{\delta-1}\theta\sigma}{(r+\sigma)} \frac{R^*}{\left[\frac{(1-\theta)MT^\delta}{r} + \frac{\theta MT^{\delta-1}}{r+\sigma} \right]} = \frac{\theta\sigma}{(r+\sigma)} \frac{R^*}{\left[\frac{(1-\theta)MT}{r} + \frac{\theta}{r+\sigma} \right]} = \frac{\theta\sigma R^*}{\frac{(r+\sigma)(1-\theta)MT}{r} + \theta} \end{aligned} \quad (B-18)$$

Rearrange (B-18), we get:

$$\theta\sigma R^* = [\theta r + (1 - \theta)(r + \sigma)MT]L \quad (17)$$

Corollary 1

This is derived from Equation (16) when assuming MT is fixed. This is a partial equilibrium analysis.

Corollary 2

This is from the definition of the meeting rates and the balance of the steady state flows. The inverse of the meeting rates has the time unit. When R is a random variable, the corresponding expected length of each agent staying in the searching state is

$\frac{1}{\alpha_f[1-F(R^*)]}$ for private firms, $\frac{1}{\alpha_b[1-F(R^*)]}$ for investment banks.

